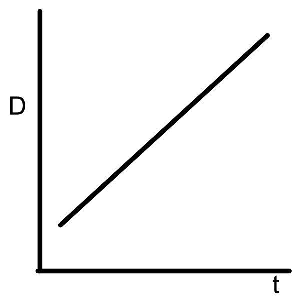
Qualitative Motion Graph Interpretations

Qualitative motion graphs are representations that focus on the important general features of motion. Looking at qualitative graphs helps us to make sense of a situation and allows us to make predictions and draw conclusions. In this way, even a simple qualitative graph can communicate a great deal of information. When analyzing motion graphs qualitatively we are not concerned with specific values, but instead focus on the essence of the motion.

There are 3 main factors we look at: Direction, Velocity, and Acceleration. We call this the DVA analysis. The analysis should always follow the following format:



D:

V:

A:

What information is needed to fill out the DVA analysis is on the following pages (Graphical Interpretations Flow Chart).

The connections between position, velocity, and acceleration formed one of the important themes of differential calculus. We will find that these relationships also form an important application of the definite integral, especially in cases in which one of the quantities varies with time.

We find that there is a direct relationship between the position function and the velocity function. By analyzing a position function or graph we are able to predict what the velocity function or graph would be.

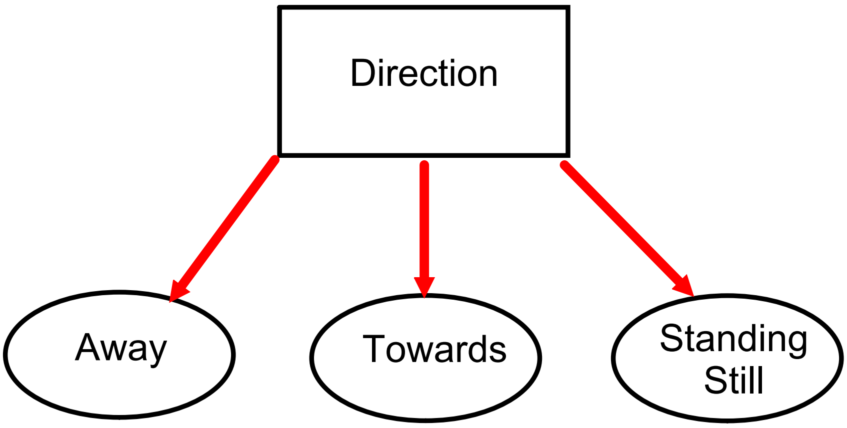
Similarly, We find that there is a direct relationship between the velocity function and the acceleration function. By analyzing a velocity function or graph we are able to predict what the acceleration function or graph would be.

However, the position function and the acceleration function are NOT directly related. We cannot directly determine any information about acceleration simply by looking at position and we cannot directly determine any information about position simply by looking at acceleration. To determine any information we must first analyze velocity and then the next quantity. Thus, position and acceleration are indirectly related.

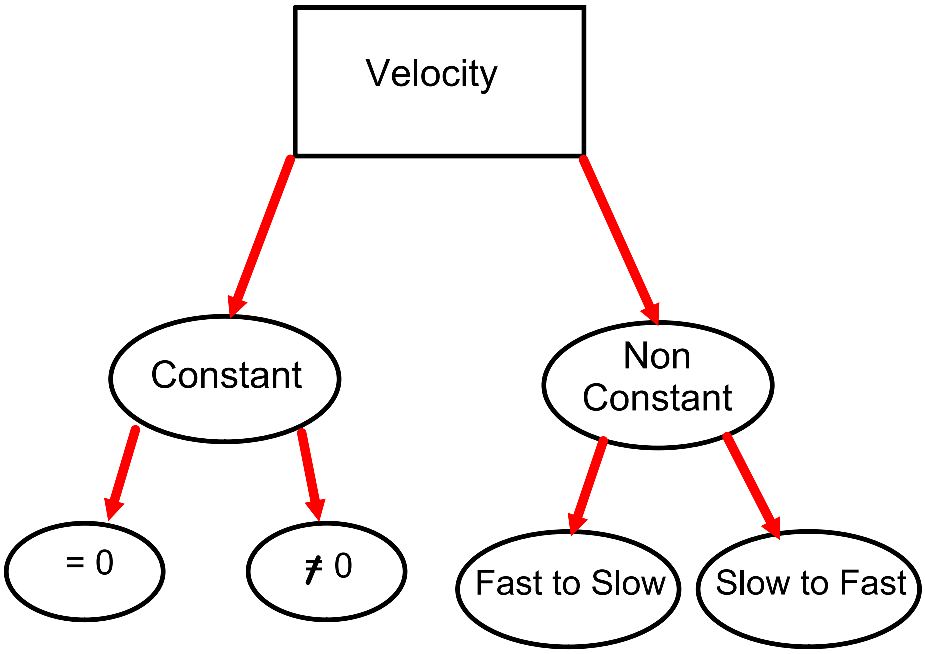
Important Concepts

1. **Acceleration**: The rate of change of velocity with respect to time in a specific direction, the derivative of velocity with respect to time.
2. **Curve**: A word used to indicate any path, whether actually curved or straight, closed or open. **For this topic a curve will NOT represent a line but will indicate a shape where the slope is NOT constant or varies.** (Circles, arcs, and parabolas)
3. **Displacement**: The difference between the initial position of something (as a body or geometric figure) and any later position. Change in position.
4. **Line**: The geometric figure formed by two points. A line is the straight path connecting two points and extending beyond the points in both directions. Follows the equation of a line (y = mx + B) where the slope along the line is a constant.
5. **Velocity**: The rate of change of position in a specific direction with respect to time, the derivative of position with respect to time.

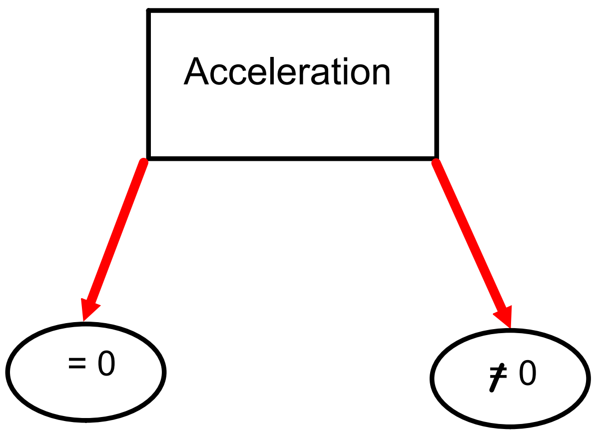
Graphical Interpretations Flow Chart



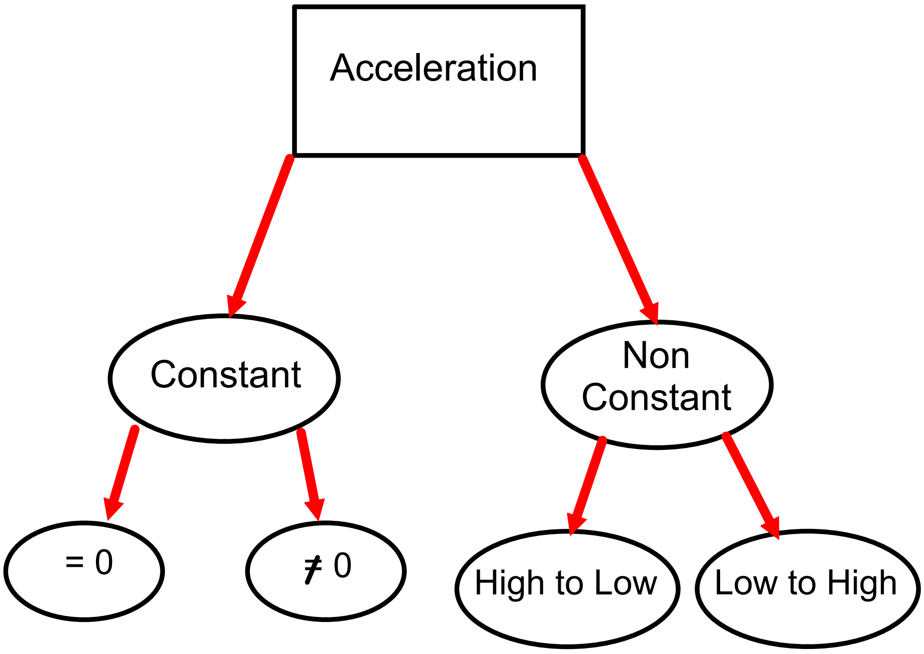
Graphical Interpretations Flow Chart Continued



Graphical Interpretations Flow Chart Continued
Position Graphs ONLY



Graphical Interpretations Flow Chart Continued
Velocity and Acceleration Graphs

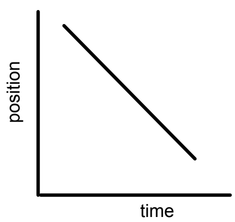


Motion Graph Analysis
When to seperate the graph into multiple points

Motion graphs may represent a simple or very complex motion. Because of this fact, we must take great care to be fully descriptive in our analysis. To do this we look for consistent motion segments. A consistent motion segment is a segment where the 3 aspects of motion (Direction - Velocity - Acceleration) do not change.

- Direction does not change
  - > Always away
  - > Always towards
  - > Always standing still
- Velocity always is doing the same thing.
  - > Always constant = 0
  - > Always constant ≠ 0
  - > Always non-constant fast to slow
  - > Always non-constant slow to fast
- Acceleration always is doing the same thing.
  - > Always constant = 0
  - > Always constant ≠ 0
  - > Always non-constant high to low
  - > Always non-constant low to high

For a simple motion graph the entire graph may be consistent, in this case the graph would NOT be separated into segments. See the graph below:

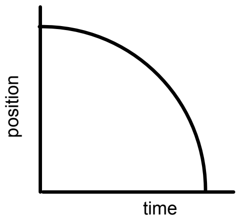


For this graph:
- direction was consistent
- velocity was consistent
- acceleration was consistent

Thus, the graph is not separated into segments.

Motion Graph Analysis
When to seperate the graph into multiple points

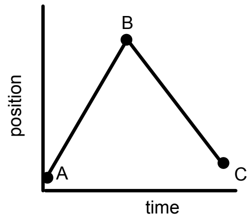
Another example where the entire graph is consistent:



For this graph:
- direction was consistent
- velocity was consistent
- acceleration was consistent

Thus, the graph is not separated into segments.

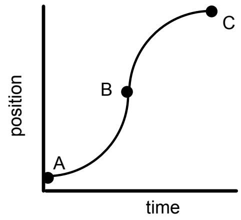
An example where the entire graph is **NOT** consistent:



For this graph:
- direction was **not** consistent
- velocity was consistent
- acceleration was consistent

Thus, the graph is separated into segments that are consistent.

Another example where the entire graph is **NOT** consistent:



For this graph:
- direction was consistent
- velocity was **not** consistent
- acceleration was consistent

Thus, the graph is separated into segments that are consistent.

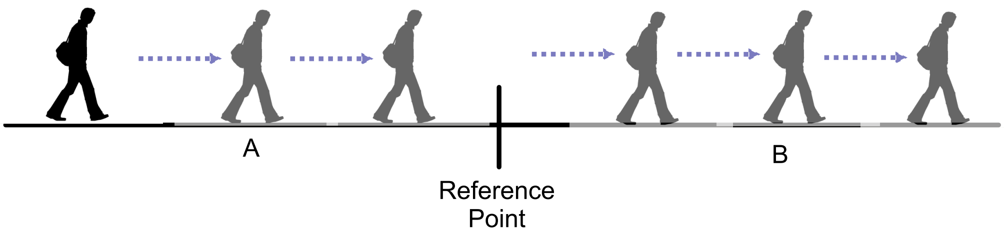
What is Direction?

Direction is a relative description for motion. It can **ONLY** be determined by comparing the motion to some fixed point in a given frame of reference.

Consider the figure below, where the reference point is placed in the middle of a line. A student is walking to the right.

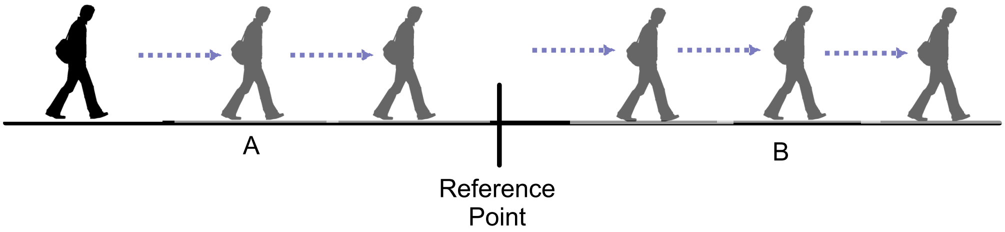
When the student is on side A moving right, he is getting closer to the reference point, and is therefore walking ***towards*** the reference point.

When the student is on side B moving right, he is getting farther away from the reference point, and is therefore walking ***away*** from the reference point.



Even though the student was always walking to the right the direction of motion changed as he passed the reference point.

What is Direction?



The direction **towards** means the object is getting closer to the reference point.

The direction **away** means the object is moving farther from the reference point.

Position Graph Analysis
Direction

In order to determine the direction of motion from a position graph:

1) Find the time axis, this is the reference point in most cases.
2) Look at the end points of the motion segment.

If the initial point is closer to the time axis then the end point, then the graph has a positive inclination and the direction of motion is AWAY.

If the initial point is farther from the time axis then the end point, then the graph has a negative inclination and the direction of motion is TOWARDS.

If the line is horizontal then there is no inclination and the direction of motion is STANDING STILL.

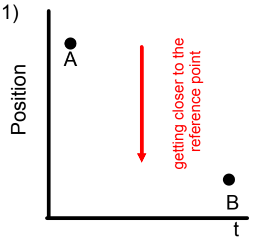
How the points are connected DOES NOT influence direction!

Calculus Method

Use the first derivative test to determine if the function has a positive or negative inclination.

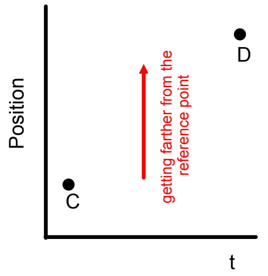
When the first derivative is positive (> 0) then the direction is AWAY.
When the first derivative is negative (< 0) then the direction is TOWARDS.
When the first derivative is zero (= 0) then the direction is STANDING STILL.

1)



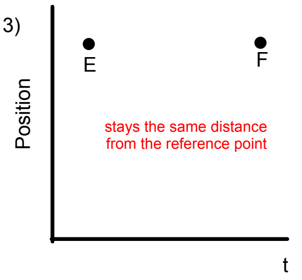
Point A is father from the time axis than point B, therefore the direction is **TOWARDS**

2)



Point D is father from the time axis than point C, therefore the direction is **AWAY**

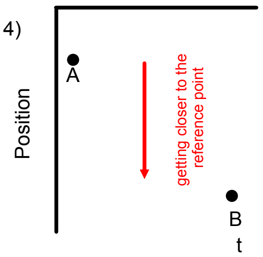
3)



Point E is the same distance from the time axis as point F, therefore the direction is **STANDING STILL**

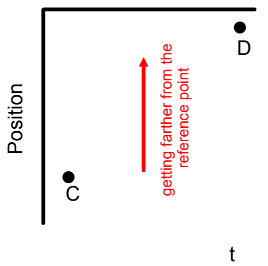
Consider walking into the negative distance domain, the idea of away and towards are a bit different.

4)



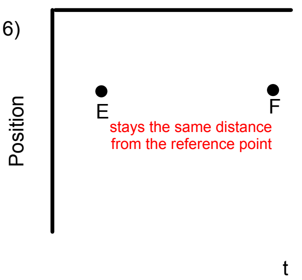
Point A is father from the time axis than point B, therefore the direction is **AWAY**

5)



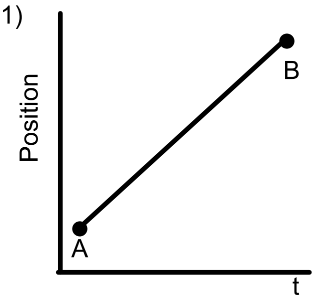
Point D is father from the time axis than point C, therefore the direction is **TOWARDS**

6)



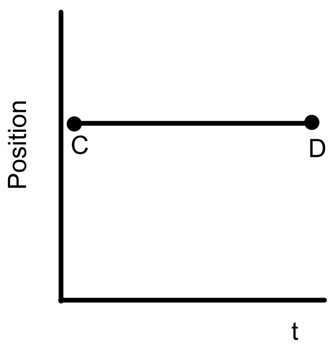
Point E is the same distance from the time axis as point F, therefore the direction is **STANDING STILL**

1)



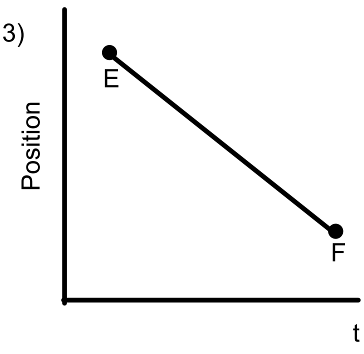
AWAY

2)



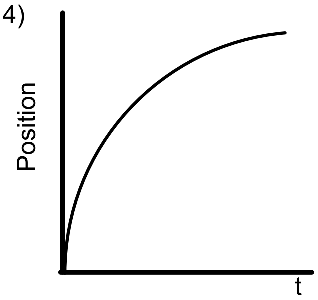
STANDING STILL

3)



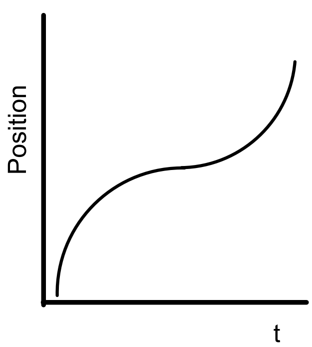
TOWARDS

4)



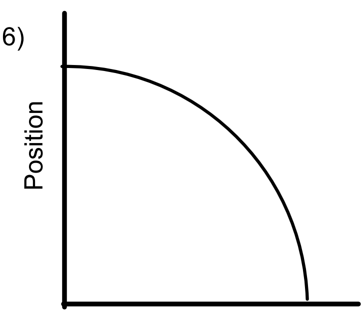
AWAY

5)



AWAY

6)

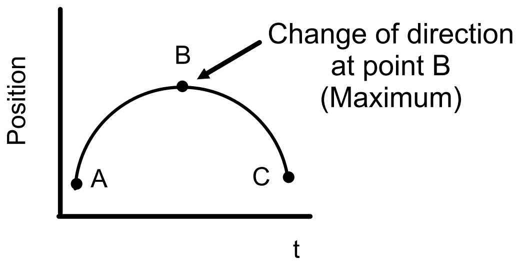


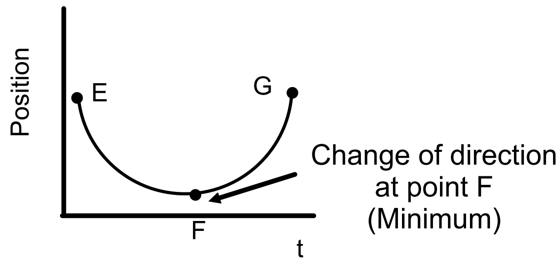
TOWARDS t

Position Graph Analysis
Direction

Change of Direction

A change of direction occurs when an objects stops going away and starts going towards, or stops going towards and starts going away. When a change of direction occurs it forms a peak or a valley on a position graph. Mathematically these are called local maximums and minimums.





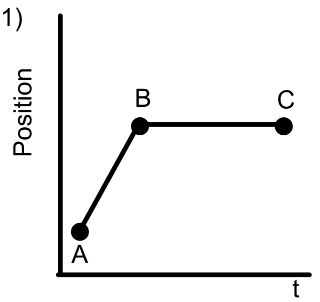
Direction change can occur at a point as in the examples above, but they can also happen along a line segment. **NOTE** a direction change does not occur just because an object stops, the object must start moving again in the opposite direction for there to be a direction change.

Position Graph Analysis
Direction

Direction Change

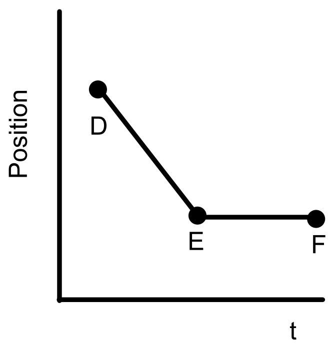
A change of direction does not occur just because an object has stopped. Standing still simply is a boundary between walking away or towards. The examples below show a an object in motion where there is no change in direction.

1)



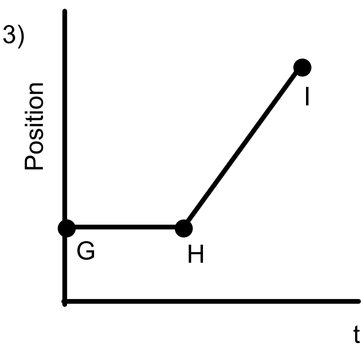
A-B motion is away, and B-C motion stops (standing still) - Direction did **NOT** change

2)



D-E motion is towards, and E-F motion stops (standing still) - Direction did **NOT** change

3)



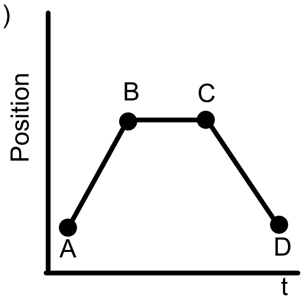
G-H motion is stopped (standing still), and H-I motion is away - Direction did **NOT** change

Position Graph Analysis
Direction

Direction Change

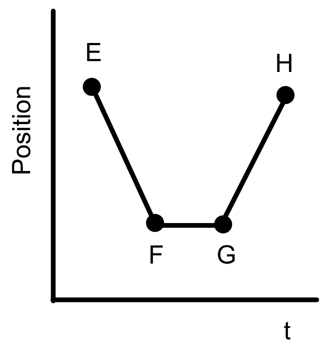
The examples 1 and 2 below show a an object in motion with a direction change. Example 3 shows an object in motion without a direction change.

1)



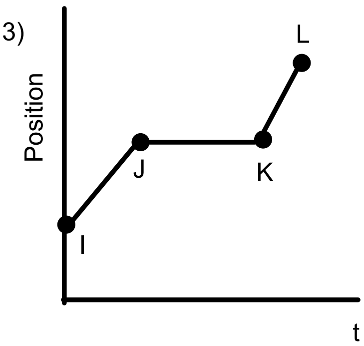
A-B motion is away, B-C motion stops (standing still), and D-E motion is towards- Direction changed at segment B-C

2)



E-F motion is towards, F-G motion stops (standing still), and G-H is away- Direction changed at segment F-G

3)



I-J motion is away, J-K motion stops (standing still), and K-L motion is away - Direction did **NOT** change

Position Graph Analysis
Velocity

Slope on a postion graph is calculated by taking the rise and dividing it by the run (rise over run). The rise on a position graph is change in position measured in meters, the run is time measured in seconds. The slope is then:

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{change in position}}{\text{time}} = \text{velocity}$$

$$\text{units for Slope} = \frac{m}{s}$$

Slope on a position Graph is Velocity

Position Graph Analysis
Velocity

Slope on a position graph is Velocity

In order to determine the velocity of motion from a position graph:

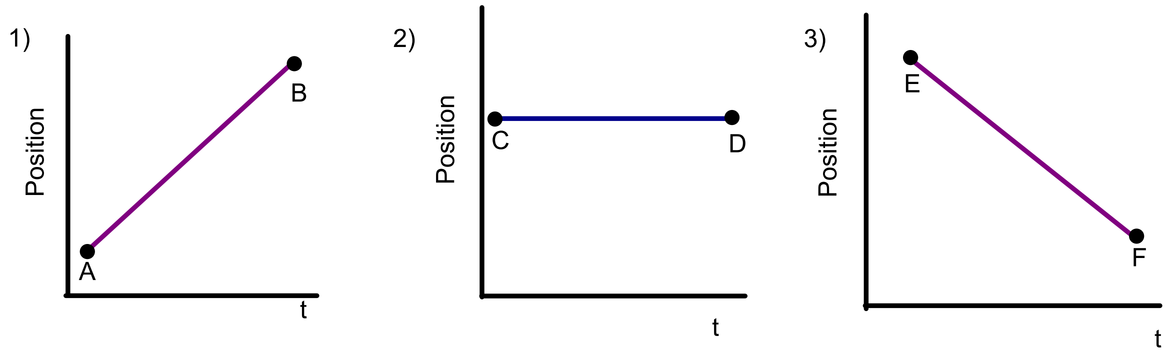
Look at how the points are connected. (Straight Line or Curve)

- **Straight Lines** - Since straight lines have constant slope; a straight line on the position graph indicates **constant velocity**.
- **Curves** - Since Curves have non-constant or varying slope; a curve on the position graph indicates **non-constant velocity**.

Note: Vertical lines cannot exist on a position graph or any other motion graphs.

Position Graph Analysis
Velocity

Constant velocity =0 or ≠ 0



All 3 graphs above are lines, therefore all 3 graphs show that velocity is constant. However, the constant velocities are different. According to the motion description flow chart, we must differentiate between zero and non-zero constant velocities.

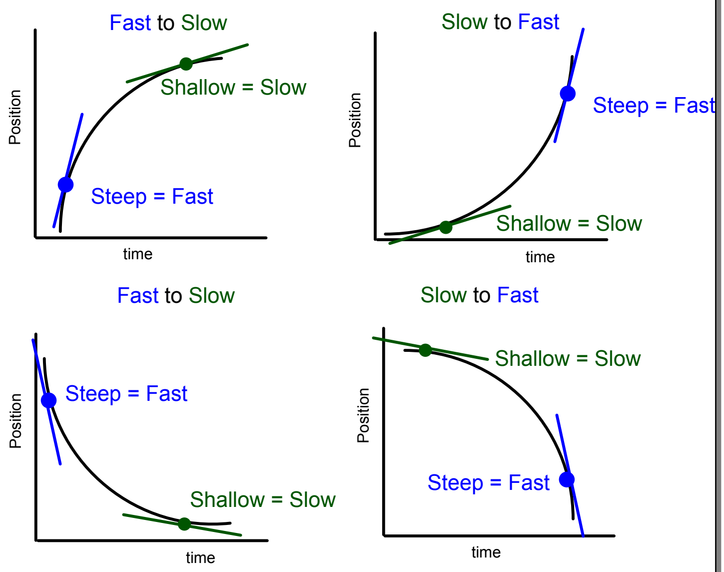
- **Horizontal lines** have a slope equal to zero, therefore, horizontal lines show a constant velocity that is **equal to zero**.
- **Diagonal lines** have a slope not equal to zero, therefore, diagonal lines show a constant velocity that is **not equal to zero**.

Position Graph Analysis
Velocity

Non constant velocity is a curve on a position graph. How can we determine if the velocity is fast or slow? The answer is curvature profile. Since slope on a position graph is velocity, we look at the slope at the endpoints of the curve. Imagine a tangent line drawn at the end point of a curve (Remember the slope of the tangent line is the slope of the curve at that point)

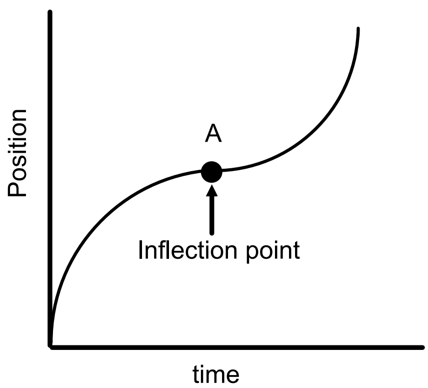
Draw tangent lines at the end points of the curve.

- If the tangent line has a shallow (horizontal) slope, then the velocity is slow.
- If the tangent line has a steep (more vertical) slope, then the velocity is fast.

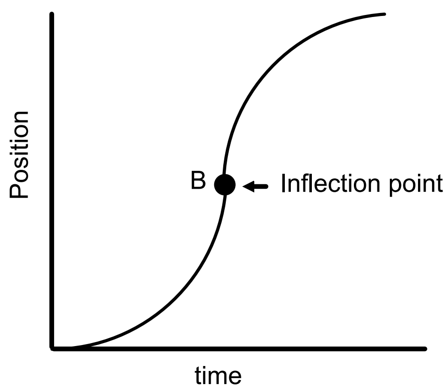


Inflection Points

In differential calculus, an inflection point, point of inflection, flex, or inflection (inflexion) is a point on a curve at which the curvature or concavity changes sign from plus to minus or from minus to plus. The curve changes from being concave upwards (positive curvature) to concave downwards (negative curvature), or vice versa. An inflection point is **NOT** a change of direction. In calculus a point of inflection can be found using the second derivative test.



At point A the object stops slowing down and starts speeding up.



At point B the object stops speeding up and starts slowing down.

Position Graph Analysis
Acceleration

Since, the position function and the acceleration function are **NOT** directly related. There is no direct way to determine any information about acceleration simply by looking at position graph. Any information about acceleration must be indirectly determined by analyzing velocity. Position graphs are related to acceleration graphs **THROUGH** velocity.

The ONLY description of acceleration for position graphs is to determine if acceleration is equal to zero or if it is not equal to zero.

For Position Graphs: Do NOT say acceleration is constant or non constant! Do NOT say it is high to low or low to high!

Analyze the position graph to determine if velocity is constant or non constant.

- If velocity is constant (equal to anything), LINE, then acceleration is equal to zero.
- If velocity is non constant, CURVE, then acceleration is **not** equal to zero.

Line on a position Graph
(Constant Velocity)

$$s(x) = mx + B$$ Equation of a line

$$V(x) = s'(x) = m$$ Constant

$$A(x) = V'(x) = s''(x) = 0$$

$$J(x) = A'(x) = V''(x) = s'''(x) = 0$$

Parabolic Curve on a position Graph (Constant Acceleration)

$s(x) = Ax^2 + Bx + C$ Equation of a Parabola

$V(x) = s'(x) = 2Ax + B$ Equation of a line

$A(x) = V'(x) = s''(x) = 2A$ Constant

$J(x) = A'(x) = V''(x) = s'''(x) = 0$

Non-Parabolic Curve on a position Graph (Non-Constant Acceleration)

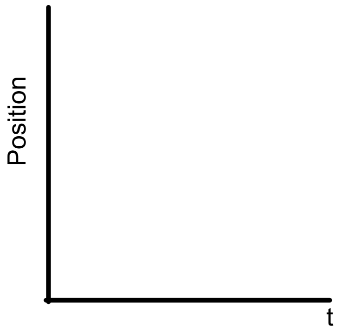
$s(x) = Ax^3 + Bx^2 + Cx + D$

Equation of a Parabola $V(x) = s'(x) = 3Ax^2 + 2Bx + C$

Equation of a line $A(x) = V'(x) = s''(x) = 6Ax + 2B$

Constant $J(x) = A'(x) = V''(x) = s'''(x) = 6A$

Practice Problems



D:

V:

A:

Away
Away
Towards
Towards

Standing Still

Non Const
Non Const
Non Const
Non Const
Non Const
Non Const
Non Const
Non Const

Const
Const
Const
Const
Const
Const
Const
Const

= 0
= 0
= 0
= 0
not = 0
not = 0
not = 0
not = 0

Fast to Slow
Fast to Slow
Fast to Slow
Fast to Slow

Slow to Fast
Slow to Fast
Slow to Fast
Slow to Fast

Low to High
Low to High
Low to High
Low to High

High to Low
High to Low
High to Low
High to Low

AB

BC

Velocity Graph Analysis
Direction

On a velocity graph there is no way of determining a reference point. All that is shown is the objects velocity at any point in time. An exact position can NOT be determined! So direction, as velocity and acceleration, has a different set of rules than the position graphs. Do **NOT** use the rules for position graph analysis to analyze velocity graphs.

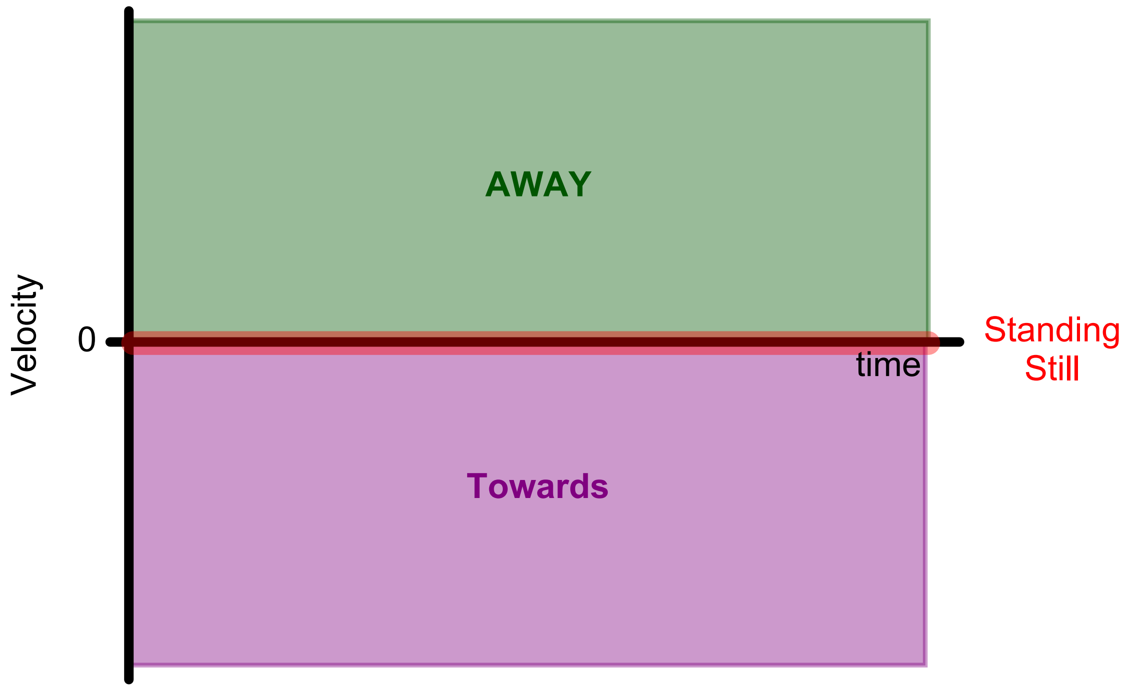
When determining Direction from a velocity graph:

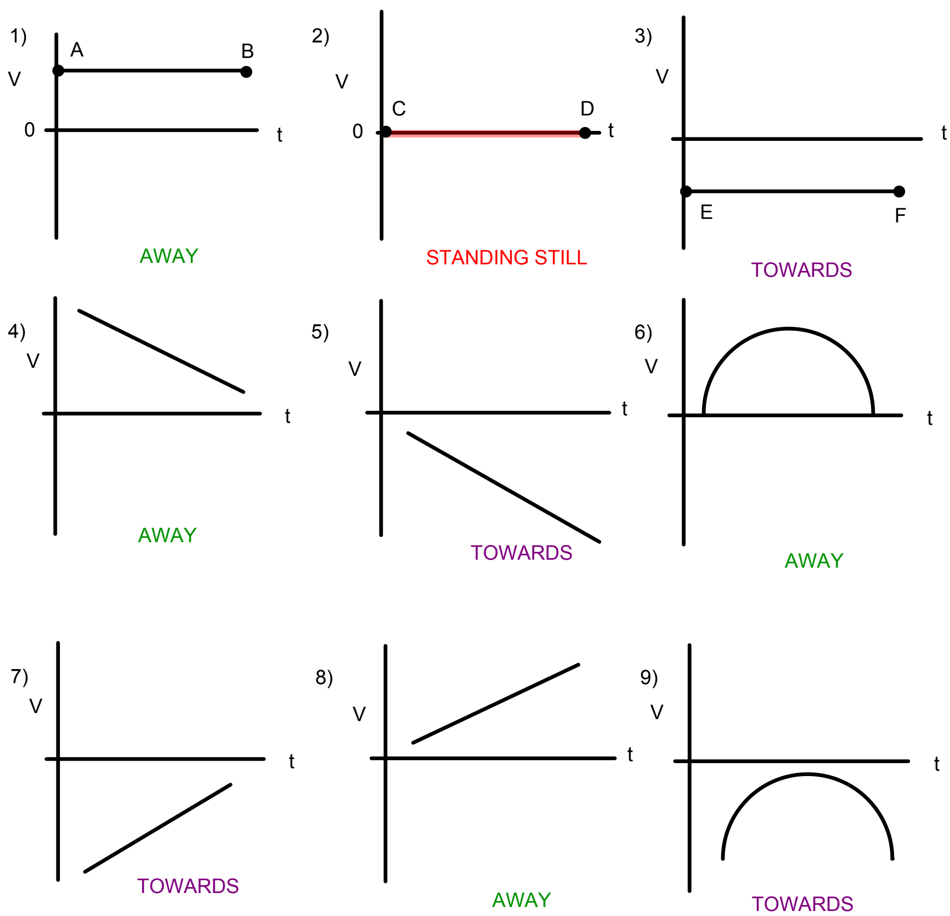
- Slope does **NOT** matter.
- Shape does **NOT** matter.

Where the points (lines or curves) are found determines the direction. (Note: the following rule only applies when motion is in the positive distance domain, which is the standard situation)

- If the points are above the time axis, then the direction is AWAY.
- If the points are below the time axis, then the direction is Towards.
- If there is a horizontal line ON the time axis, the direction is standing still.

Velocity Graph Analysis
Direction

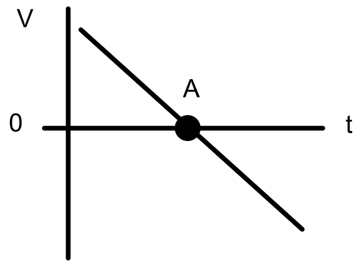




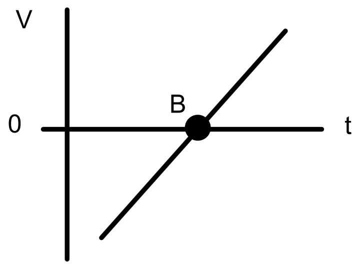
Velocity Graph Analysis
Direction

Change of Direction

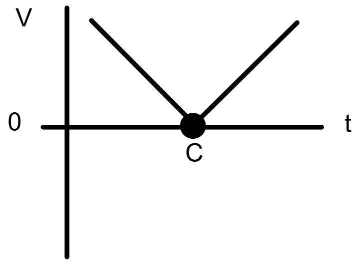
A change of direction occurs when an objects stops going away and starts going towards, or stops going towards and starts going away. When a change of direction occurs on a velocity graph the shape intersects the time axis and goes from the positive domain to the negative domain or vice versa. Note: The curve must completely pass **through** the time axis and not just touching it, for direction to have changed.



Direction changed away to towards



Direction changed towards to away



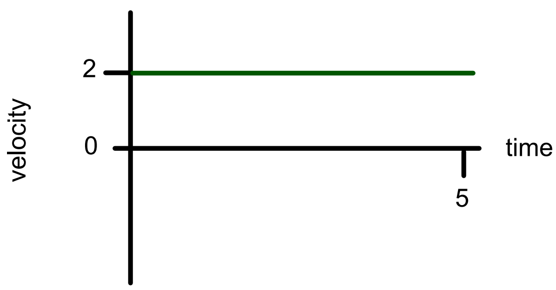
Direction did NOT change

Velocity Graph Analysis
Velocity

Velocity graphs show the velocity at every moment in time. They are graphs of the instantaneous velocities. This should make analyzing velocity very simple.

Constant vs. Non-constant Velocity

If velocity is constant, then it never changes. Which means the velocity is the same at all times. Let's say velocity is constant and equals 2 m/s for 5 s. If we were to look at this on a velocity graph it would look like this:



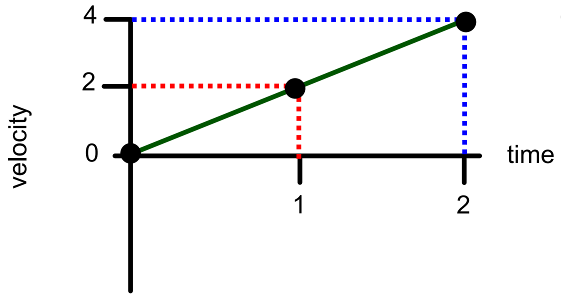
Horizontal line

Constant Velocity is a Horizontal Line on a velocity graph.

Velocity Graph Analysis
Velocity

Constant vs. Non-constant Velocity

Since constant velocity is a horizontal line, what does non-constant velocity look like? Look at the graph below:



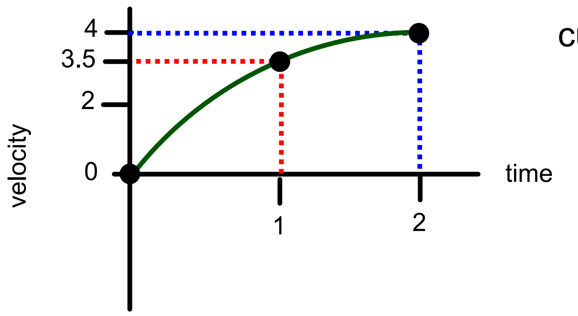
diagonal line

At 0 s the velocity is 0, at 1 s the velocity is 2 m/s, and at 2 s the velocity is 4 m/s. Since the values for the velocities are not the same, velocity must be non-constant. Therefore, **diagonal lines show that velocity is non-constant.**

Velocity Graph Analysis
Velocity

Constant vs. Non-constant Velocity

Are diagonal lines the only shape that show non-constant velocity? Look at the graph below:



curve

At 0 s the velocity is 0, at 1 s the velocity is 3.5 m/s, and at 2 s the velocity is 4 m/s. Since the values for the velocities are not the same, velocity must be non-constant. Therefore, **curves show that velocity is non-constant.**

Velocity Graph Analysis
Velocity

Constant vs. Non-constant Velocity

Constant Velocity Looks like a horizontal line on a velocity graph.

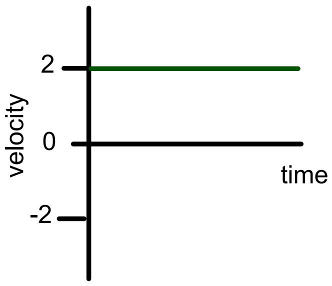
Non-Constant Velocity looks like a diagonal line or a curve on a velocity graph.

Note: Vertical lines cannot exist on a velocity graph or any other motion graphs.

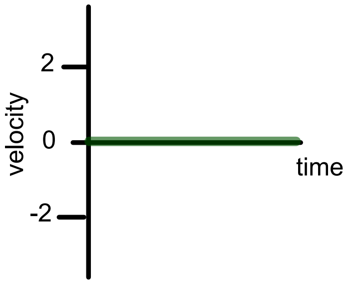
Velocity Graph Analysis
Velocity

Constant Velocity

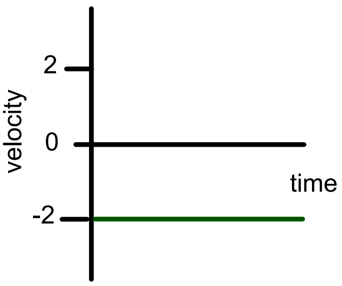
Now that we know constant velocity looks like a horizontal line on a velocity graph, we need to determine whether the constant velocity is equal to zero or not equal to zero. This is extremely simple to determine. Since these are velocity graphs, all that is needed is to **read the position of the horizontal line** from the velocity axis. All 3 graphs below show constant velocity.



Horizontal line at 2 m/s
constant velocity = 2 m/s
constant velocity ≠ 0



Horizontal line at 0 m/s
constant velocity = 0 m/s
constant velocity = 0

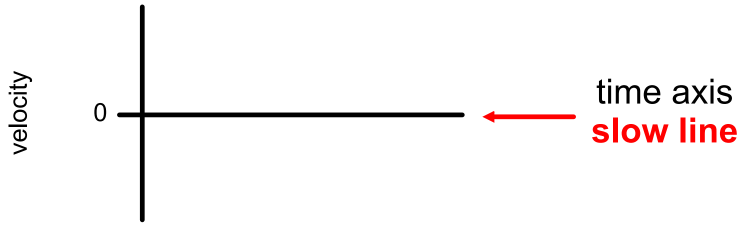


Horizontal line at -2 m/s
constant velocity = -2 m/s
constant velocity ≠ 0

Velocity Graph Analysis
Velocity

Non-constant Velocity

We know non-constant velocity looks like a diagonal line or a curve on a velocity graph, we need to determine whether the non-constant velocity is fast to slow or slow to fast. Slow and fast are relative terms, and the use of negative and positive signs may add to the confusion. Unlike in mathematics, the sign on velocity indicates the direction of motion, so 5 m/s is **not** faster than - 5 m/s. In fact they are the same speed just in opposite directions. So we need something else to base these relative terms on. Let us then consider the term slow, what is the limit of slowness? How slow can you go? That's easy, the limit of slowness is not moving at all, to have a velocity equal to zero. Zero velocity is the time axis on the velocity graph, therefore, we will rename the time axis on the velocity graph to be called the **slow line**.

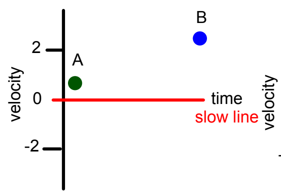


Velocity Graph Analysis
Velocity

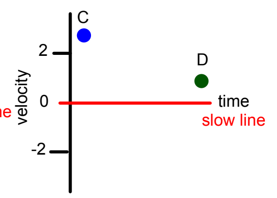
Non-constant Velocity

To determine if motion is slow to fast or fast to slow we must analyze 2 points, one at the beginning and on at the end of a unique shape. **How these points are connected DOES NOT MATTER!**

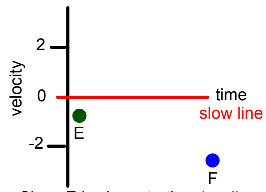
- Points that are closer to the slow line are the slow velocities.
- Points that are farther from the slow line are the fast velocities.



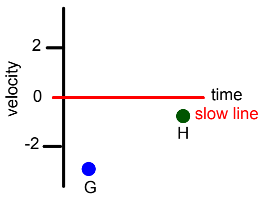
Since A is closer to the slow line A is slow while B is fast, Thus velocity is non-constant slow to fast.



Since D is closer to the slow line C is fast while D is slow, Thus velocity is non-constant fast to slow.



Since E is closer to the slow line E is slow while F is fast, Thus velocity is non-constant slow to fast.



Since H is closer to the slow line G is fast while H is slow, Thus velocity is non-constant fast to slow.

Velocity Graph Analysis
Velocity

Constant Velocity Looks like a horizontal line on a velocity graph.

- **= 0 when the horizontal line is ON the time axis.**
- **≠ 0 When the horizontal line is NOT ON the time axis.**

Non-Constant Velocity looks like a diagonal line or a curve on a velocity graph.

- **Slow to Fast when the shape starts close to the time axis and finishes farther from the time axis.**
- **Fast to Slow when the shape starts away from the time axis and finishes closer to the time axis.**

Velocity Graph Analysis
Acceleration

Velocity and acceleration are directly related, therefore a more detailed analysis of acceleration is needed than was used when analyzing a postion graph.

Slope on a velocity graph is calculated by taking the rise and dividing it by the run (rise over run). The rise on a velocity graph is change in velocity measured in m/s, the run is time measured in seconds. The slope is then:

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{change in velocity}}{\text{time}} = \text{acceleration}$$

$$\text{units for Slope} = \frac{m/s}{s} = \frac{m}{s^2}$$

Slope on a Velocity Graph is Acceleration

Velocity Graph Analysis
Acceleration

Slope on a Velocity graph is Acceleration

In order to determine the acceleration of motion from a velocity graph:

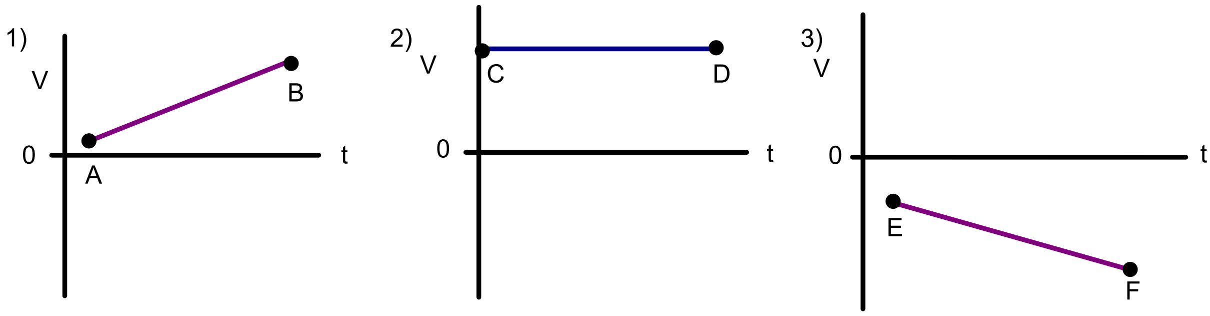
Look at how the points are connected. (Straight Line or Curve)

- **Straight Lines** - Since straight lines have constant slope; a straight line on the velocity graph indicates **constant acceleration**.
- **Curves** - Since Curves have non-constant or varying slope; a curve on the velocity graph indicates **non-constant acceleration**.

Note: Vertical lines cannot exist on a velocity graph or any other motion graphs.

Velocity Graph Analysis
Acceleration

Constant Acceleration =0 or ≠ 0



All 3 graphs above are lines, therefore all 3 graphs show that acceleration is constant. However, the constant accelerations are different. According to the motion description flow chart, we must differentiate between zero and non-zero constant accelerations.

- **Horizontal lines** have a slope equal to zero, therefore, horizontal lines show a constant acceleration that is **equal to zero**.
- **Diagonal lines** have a slope not equal to zero, therefore, diagonal lines show a constant acceleration that is **not equal to zero**.

Velocity Graph Analysis
Acceleration
high vs. low

When acceleration is non-constant we must determine if the acceleration is high or low. But what does high and low refer to?

high accelerations occur when the velocity is **changing a lot**.

low accelerations occur when the velocity is **changing a little**.

See the tables below:

Table 1

time	1 s	2 s	3 s	4 s	5 s	6 s	7 s
velocity	0 m/s	10 m/s	18 m/s	24 m/s	28 m/s	30 m/s	31 m/s

High **Low**

The object is always speeding up non-uniformly. The velocity changed a lot during the 1 s to 3 s interval, but only a little during the 5 s to 7 s interval. Therefore, from 1 to 3 s the acceleration is **high** and from 5 to 7 s the acceleration is **low**.

Table 2

time	1 s	2 s	3 s	4 s	5 s	6 s	7 s
velocity	30 m/s	20 m/s	12 m/s	6 m/s	2 m/s	1 m/s	0 m/s

High **Low**

The object is always slowing down non-uniformly. The velocity changed a lot during the 1 s to 3 s interval, but only a little during the 5 s to 7 s interval. Therefore, from 1 to 3 s the acceleration is **high** and from 5 to 7 s the acceleration is **low**.

- In table 1 the object was speeding up and in table 2 the object was slowing down.
- In table 1 high acceleration occurred when the velocity was slow, while in table 2 high acceleration occurred when the velocity was fast.
- In table 1 low acceleration occurred when the velocity was fast, while in table 2 low acceleration occurred when the velocity was slow.

Velocity Graph Analysis
Acceleration
high vs. low

High and low **DO NOT** depend on how velocity is changing.

High and low **DO NOT** correspond to fast and slow.

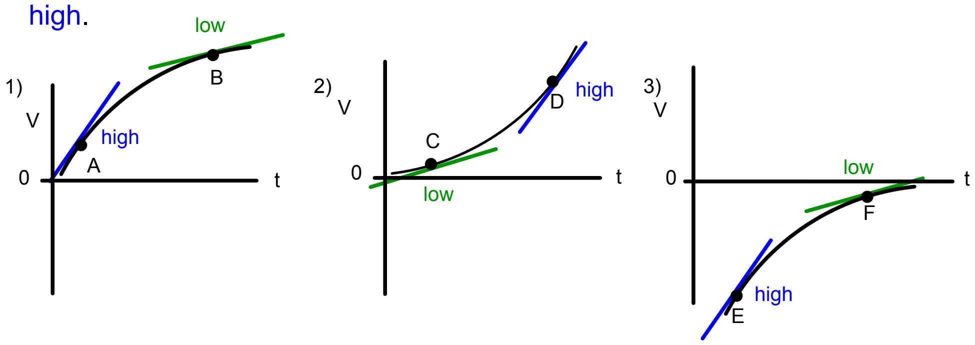
- It is possible for velocity to be fast and acceleration to be low at the same point.
- It is possible for velocity to be fast and acceleration to be high at the same point.
- It is possible for velocity to be slow and acceleration to be low at the same point.
- It is possible for velocity to be slow and acceleration to be high at the same point.

Velocity Graph Analysis
Acceleration

Non-constant acceleration is a **curve** on a velocity graph. How can we determine if the acceleration is high or low? The answer is curvature profile. Since slope on a velocity graph is acceleration, we look at the slope at the endpoints of the curve. Imagine a tangent line drawn at the end point of a curve (Remember the slope of the tangent line is the slope of the curve at that point)

Draw tangent lines at the end points of the curve.

- If the tangent line has a shallow (horizontal) slope, then the acceleration is low.
- If the tangent line has a steep (more vertical) slope, then the acceleration is high.



Velocity Graph Analysis
Acceleration

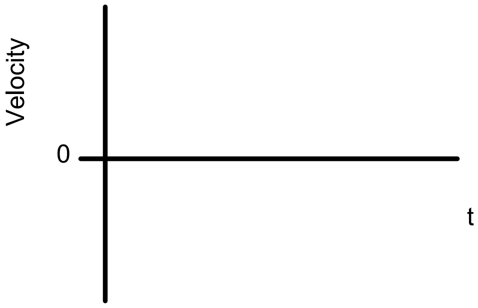
Constant Acceleration Looks like a horizontal or diagonal line on a velocity graph.

- **= 0 when the line is horizontal.**
- **≠ 0 When the line is diagonal.**

Non-Constant Acceleration looks like a curve on a velocity graph.

- **Low to High when the curve starts shallow and ends steep.**
- **High to Low when the curve starts steep and ends shallow.**

Practice Problems



D:

V:

A:

Away
Away

Towards
Towards

Standing Still

Non Const
Non Const
Non Const
Non Const
Non Const
Non Const
Non Const
Non Const

Const
Const
Const
Const
Const
Const
Const
Const

= 0
= 0
= 0
= 0
not = 0
not = 0
not = 0
not = 0

Fast to Slow
Fast to Slow
Fast to Slow
Fast to Slow

Slow to Fast
Slow to Fast
Slow to Fast
Slow to Fast

Low to High
Low to High
Low to High
Low to High

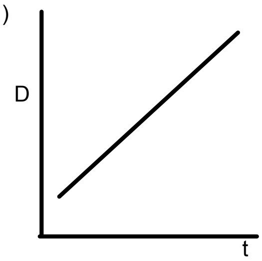
High to Low
High to Low
High to Low
High to Low

AB

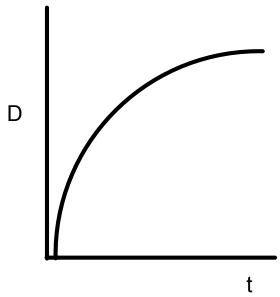
BC

Quiz

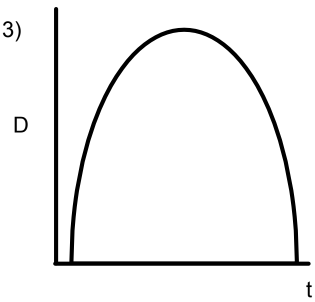
1)



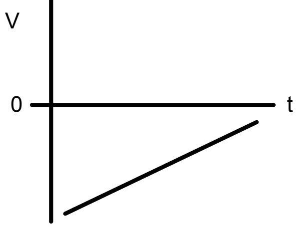
2)



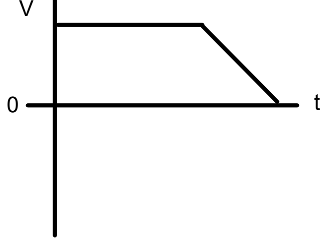
3)



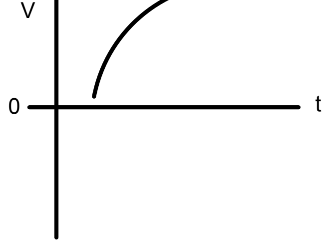
4)



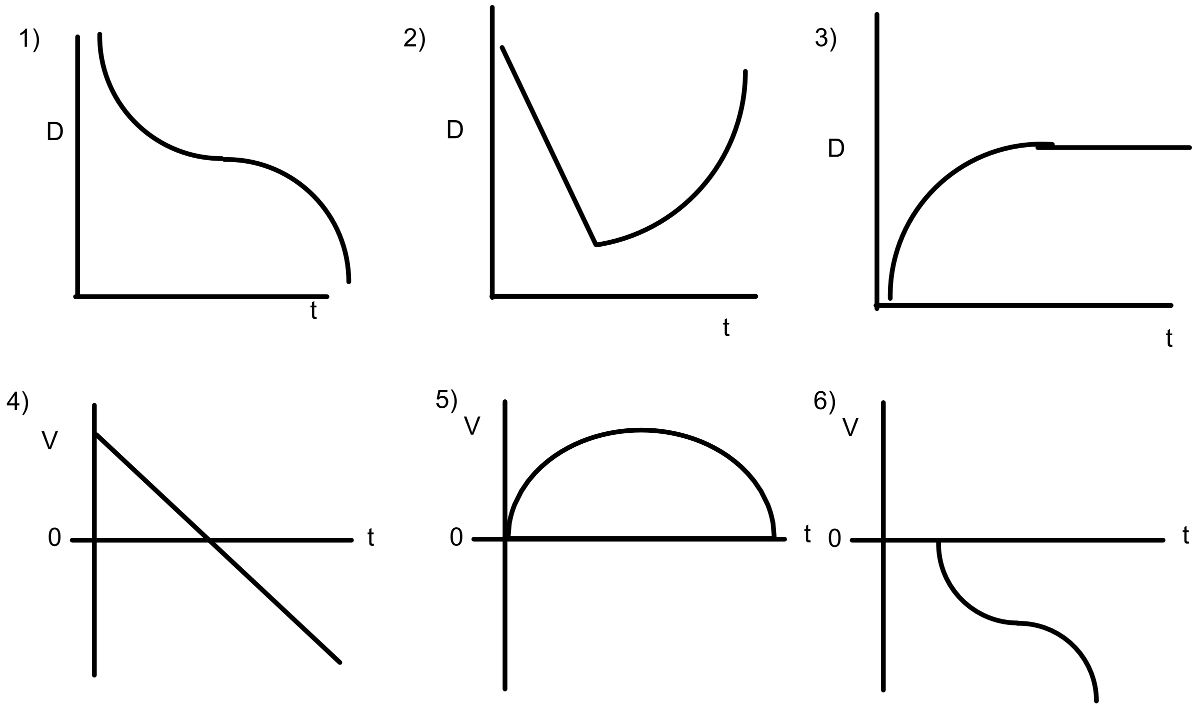
5)



6)



Quiz

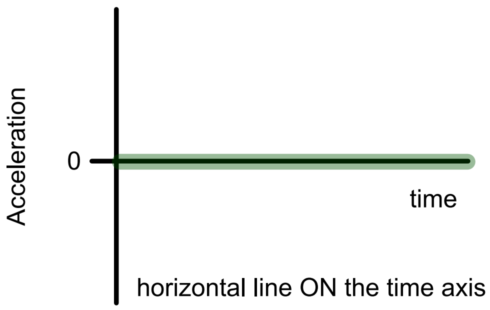


Acceleration Graph Analysis
Direction

Acceleration is **NOT** directly related to position, therefore, there is no direct way to determine direction on an acceleration graph. In fact it is impossible to determine what the true direction of motion was simply by looking at an acceleration graph. Consider the following examples:

1. A man stands still for 10 s. His velocity is constant and equal to zero.
2. A man walks away with a constant velocity of 2 m/s for 10 s.
3. A man walks towards with a constant velocity of - 2 m/s for 10 s.

In each case, velocity was constant. As we know, when velocity is constant equal to anything, acceleration is constant and equal to zero. So the acceleration graphs for each case would look like the graph below.



If all we had to analyze was this graph, how could we tell if it was referring to case 1, 2, or 3? We **COULDN'T**! So, when analyzing acceleration graphs, we must account for all the possible ways to make them.

Acceleration Graph Analysis
Direction

The question of direction still remains. To answer this we must look to velocity. Since, velocity is directly related to acceleration, the acceleration graphs should give us definite information about velocity. But it doesn't! Recall cases 1, 2, and 3 on the previous page.

- case 1 - velocity is constant equal to zero.
- case 2 - velocity is constant not equal to zero
- case 3 - velocity is constant not equal to zero

Yet the acceleration graph was the same. There was no direct way to determine what the constant velocity was equal to, simply by looking at the acceleration graph. All that is known equivocally, is that velocity is constant, equal to what? We can't tell. If direction was specified, then we could determine whether or not the constant velocity equaled zero.

In order to determine a direction, we must know something about the velocity.
In order to determine the velocity, we must know something about direction.

Acceleration Graph Analysis
Velocity

- What we can determine immediately acceleration graphs is whether velocity is constant or non-constant.

- Velocity can **ONLY** be constant if the acceleration is constant and equal to zero. **(Most importantly is the fact that acceleration equals zero)** This is seen as a horizontal line **ON** the tome axis on the acceleration graph.

- Any other shape (horizontal line **NOT ON** the time axis, diagonal line, or curve) indicates a **non-zero acceleration** and as a result a **non-constant velocity**.

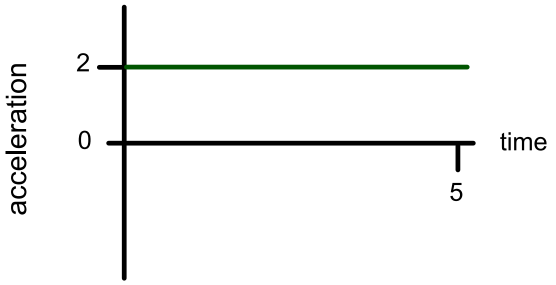
- So in analyzing the acceleration graphs we must work backwards, starting with acceleration and then moving to direction/velocity.

Acceleration Graph Analysis
Acceleration

Acceleration graphs show the acceleration at every moment in time. They are graphs of the instantaneous accelerations.

Constant vs. Non-constant Acceleration

If acceleration is constant, then it never changes. Which means the acceleration is the same at all times. Let's say acceleration is constant and equals 2 m/s^2 for 5 s. If we were to look at this on an acceleration graph it would look like this:



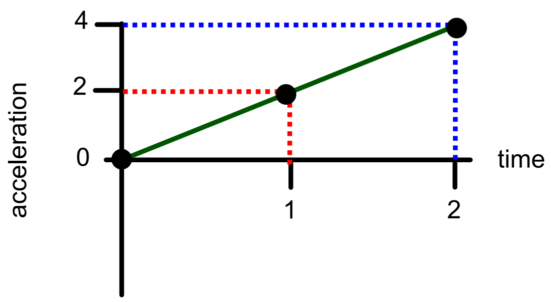
Horizontal line

Constant acceleration is a Horizontal Line on an acceleration graph.

Acceleration Graph Analysis
Acceleration

constant vs. non-constant

Since constant acceleration is a horizontal line, what does non-constant acceleration look like? Look at the graph below:



diagonal line

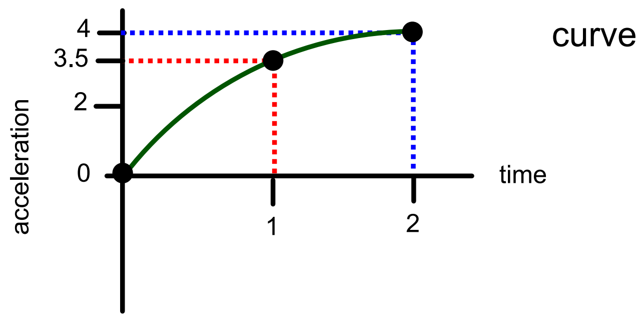
At 0 s the acceleration is 0, at 1 s the velocity is 2 m/s^2, and at 2 s the acceleration is 4 m/s^2. Since the values for the accelerations are not the same, acceleration must be non-constant. Therefore, **diagonal lines show that acceleration is non-constant.**

Acceleration Graph Analysis

Acceleration

constant vs. non-constant

Are diagonal lines the only shape that show non-constant acceleration? Look at the graph below:



At 0 s the acceleration is 0, at 1 s the acceleration is 3.5 m/s^2, and at 2 s the acceleration is 4 m/s^2. Since the values for the velocities are not the same, acceleration must be non-constant. Therefore, **curves show that acceleration is non-constant.**

Acceleration Graph Analysis

Acceleration

constant vs. non-constant

Constant Acceleration Looks like a horizontal line on an acceleration graph.

Non-Constant Acceleration looks like a diagonal line or a curve on an acceleration graph.

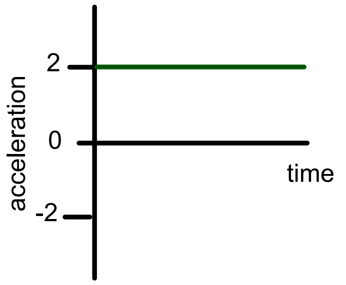
Note: Vertical lines cannot exist on an acceleration graph or any other motion graphs.

Acceleration Graph Analysis

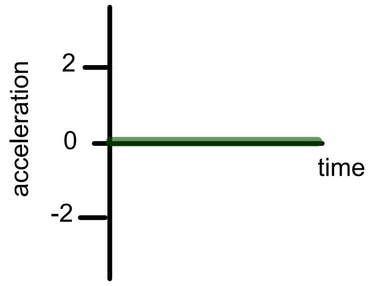
Acceleration

constant accleration = 0 or ≠ 0

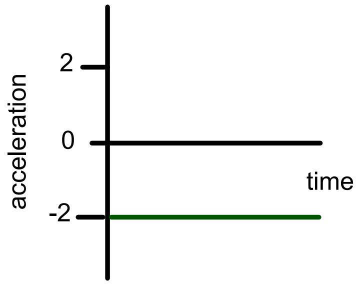
Now that we know constant acceleration looks like a horizontal line on a acceleration graph, we need to determine whether the constant acceleration is equal to zero or not equal to zero. This is extremely simple to determine. Since these are acceleration graphs, all that is needed is to **read the position of the horizontal line** from the acceleration axis. All 3 graphs below show constant acceleration.



Horizontal line at 2 m/s^2
constant acceleration = 2 m/s^2
constant acceleration ≠ 0



Horizontal line at 0 m/s^2
constant acceleration = 0 m/s^2
constant acceleration = 0



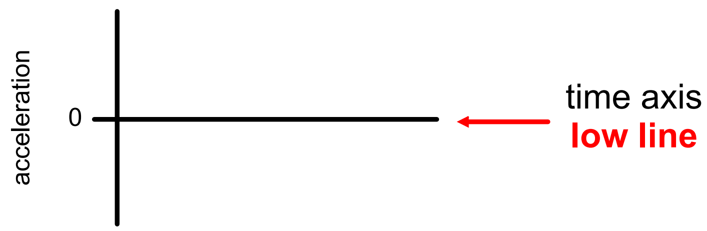
Horizontal line at -2 m/s^2
constant acceleration = -2 m/s^2
constant acceleration ≠ 0

Acceleration Graph Analysis

Acceleration

non-constant acceleration

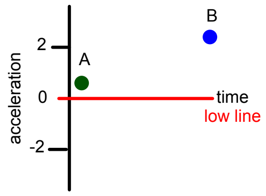
We know non-constant acceleration looks like a diagonal line or a curve on an acceleration graph, we need to determine whether the non-constant acceleration is high to low or low to high. High and low are relative terms, and the use of negative and positive signs may add to the confusion. The sign on acceleration indicates the zones of relative speed near the reference point, but the reference point is not seen on an acceleration graph. So we need something else to base these relative terms on. Let us then consider the term low, what is the lowest possible rate of acceleration? Simply, it is not accelerating at all, acceleration equal to zero. Zero acceleration is the time axis on the acceleration graph, therefore, we will rename the time axis on the acceleration graph to be called the **low line**.



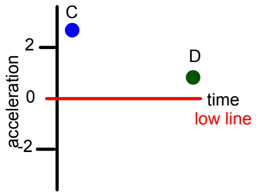
Acceleration Graph Analysis
Acceleration
non-constant acceleration

To determine if motion is low to high or high to low we must analyze 2 points, one at the beginning and on at the end of a unique shape. **How these points are connected DOES NOT MATTER!**

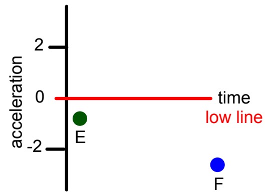
- Points is closer to the low line are the low accelerations.
- Points is farther from the low line are the high accelerations.



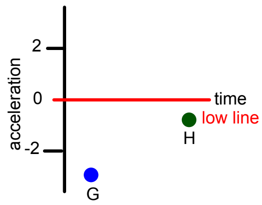
Since A is closer to the low line A is low while B is high, Thus acceleration is non-constant low to high.



Since D is closer to the low line D is low while C is high, Thus acceleration is non-constant high to low.



Since E is closer to the low line E is low while F is high, Thus acceleration is non-constant low to high.



Since H is closer to the low line H is low while G is high, Thus acceleration is non-constant high to low.

Acceleration Graph Analysis
Acceleration

Constant Acceleration Looks like a horizontal line on an acceleration graph.

- **= 0 when the horizontal line is ON the time axis.**
- **≠ 0 When the horizontal line is NOT ON the time axis.**

Non-Constant Acceleration looks like a diagonal line or a curve on an acceleration graph.

- **Low to High when the shape starts close to the time axis and finishes farther from the time axis.**
- **High to Low when the shape starts away from the time axis and finishes closer to the time axis.**

Acceleration

positive vs. negative

Acceleration is a vector quantity which is defined as the rate at which an object changes its velocity in a specific direction. An object is accelerating if it is changing its velocity or changing its direction. Acceleration is a vector quantity so it will always have a direction associated with it. The direction of the acceleration vector depends on two factors:

1. Whether the object is speeding up or slowing down.
2. Whether the object is moving in the positive or negative direction.

A common misconception is that all objects that are speeding up have positive acceleration and that all objects that are slowing down have negative accelerations. **The fact is that an object that is speeding up can have either a positive or a negative acceleration depending on the direction of the motion. Similarly, an object that is slowing down can have either a positive or a negative acceleration depending on the direction of the motion.** One must be careful not to assume the sign of an acceleration simply based on the way the speed is changing. Remember the directions away and towards are based on a reference point. The object will either move away from the reference point or towards the reference point.

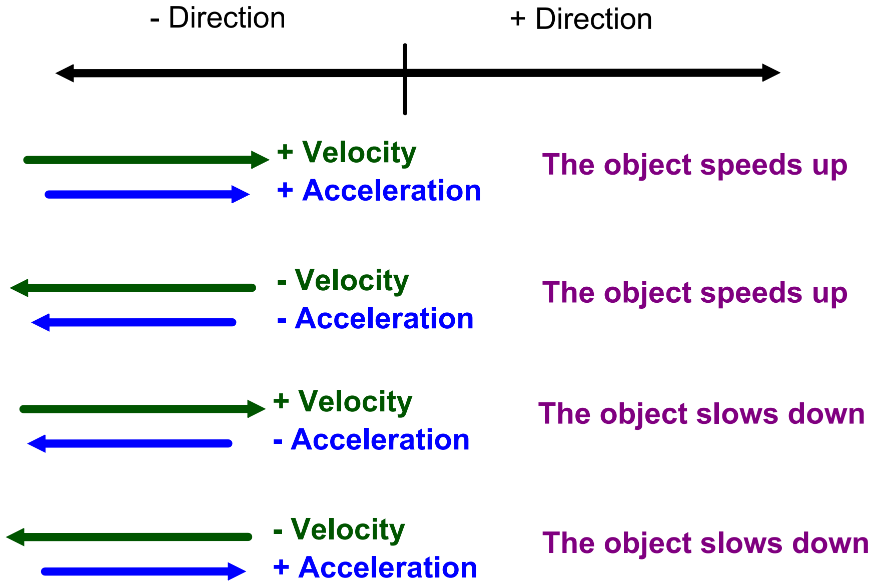
Acceleration

positive vs. negative

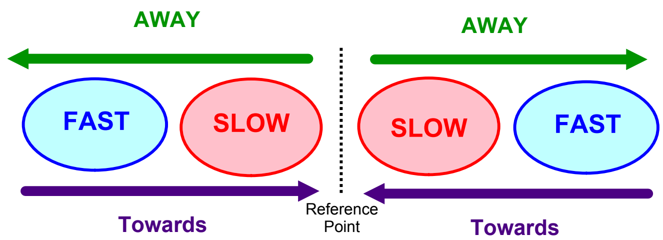
The general RULE OF THUMB is:

If an object is slowing down, then its acceleration is in the opposite direction of its motion.

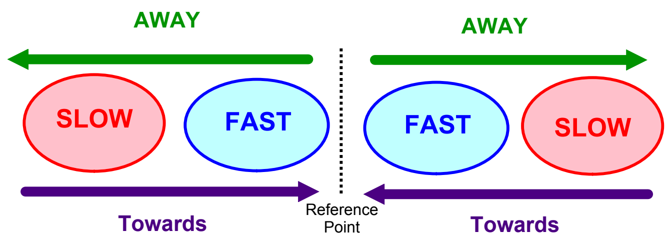
If an object is speeding up, then its acceleration is in the same direction of its motion.



Positive Acceleration



Negative Acceleration



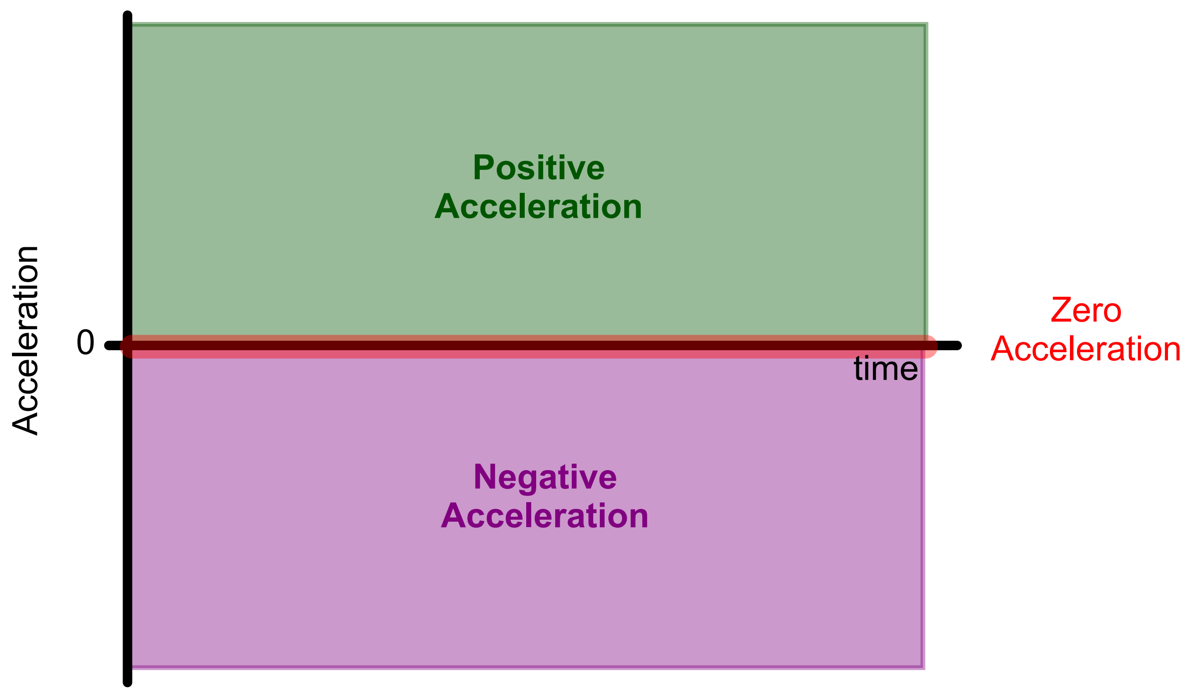
When an object is speeding up and moving away from the reference point, the acceleration is positive.

When an object is speeding up and moving towards the reference point, the acceleration is negative

When an object is slowing down and moving away from the reference point, the acceleration is negative.

When an object is slowing down and moving towards the reference point, the acceleration is positive.

Acceleration Graph Analysis
Acceleration



Acceleration Graph Analysis
Direction / Velocity
Constant Acceleration = 0

A horizontal line **ON** the time axis on an acceleration graph indicates that acceleration is constant and equal to zero, and that the **velocity is constant**. The motion descriptions (DVA) may consist of any of the following 3 cases:

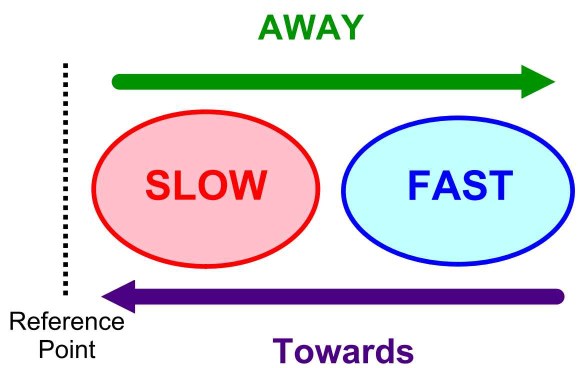
Case 1	Case 2	Case 3
D: Away	D: Towards	D: Standing Still
V: Constant $\neq 0$	V: Constant $\neq 0$	V: Constant $= 0$
A: Constant $= 0$	A: Constant $= 0$	A: Constant $= 0$

Note: This is the ONLY situation where case 3 (Standing Still) is possible!!!

We must account for ALL 3 possibilities.

Acceleration Graph Analysis
Direction / Velocity
positive acceleration

For situations when acceleration is positive (any shape in the positive domain), there will be 3 cases that apply. We know that for each case velocity will be **Non-constant**, and we must determine whether it is slow to fast or fast to slow. To determine these 3 cases we will go the cloud, the acceleration cloud. The cloud diagram for positive acceleration is shown below.

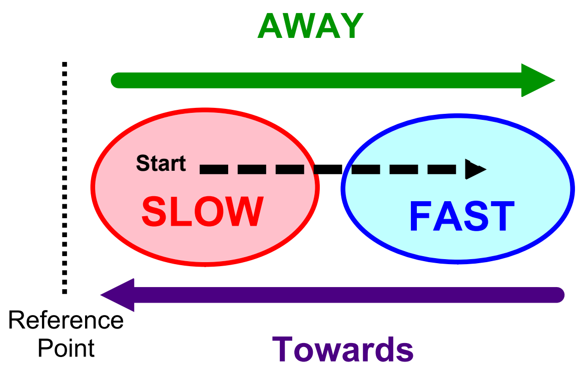


When acceleration is positive, the slow zone is near the reference point.
Note: Acceleration may be Constant ≠ 0 or Non-constant.

Acceleration Graph Analysis
Direction / Velocity

Positive Acceleration - Case 1

Choose a direction for the motion, this choice is arbitrary, but it will determine if the non-constant velocity is fast to slow or slow to fast. For case 1 the direction will be away. In order to walk away on the cloud diagram, you must start in the slow zone and move into the fast zone. Thus, the DVA description is:



D: Away

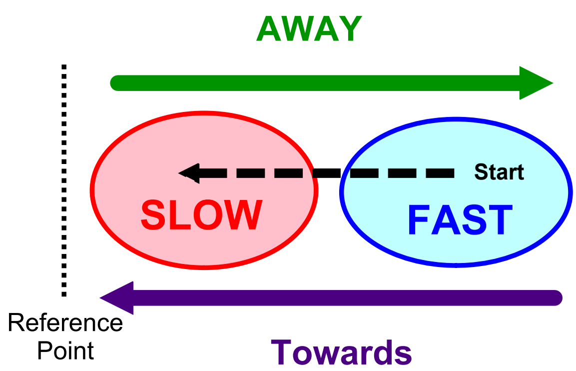
V: Non-constant
slow to fast

A: to be determined
by the shape on
the graph

Acceleration Graph Analysis
Direction / Velocity

Positive Acceleration - Case 2

Next choose the opposite direction for the motion. For case 2, the direction will be towards. In order to walk towards on the cloud diagram, you must start in the fast zone and move into the slow zone. Thus, the DVA description is:



D: Towards

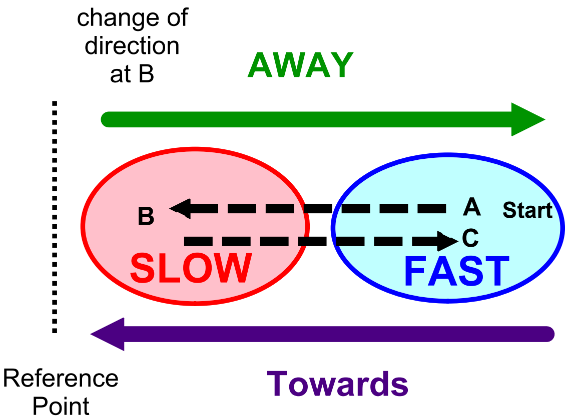
V: Non-constant
fast to slow

A: to be determined
by the shape on
the graph

Acceleration Graph Analysis
Direction / Velocity

Positive Acceleration - Case 3

No further directions for the motion can be used (Standing Still is NOT an option). For case 3, we will break the graph into an A-B-C graph, where there is a change of direction at B. We know that velocity is ZERO when direction changes, so point B must be in the slow zone. Since the slow zone is near the reference point for + accelerations we must start in the fast zone. Thus, A to B is walking towards starting in the fast zone and moving into the slow zone. Then B to C is walking away starting in the slow zone and moving into the fast zone. Thus, the DVA description is:

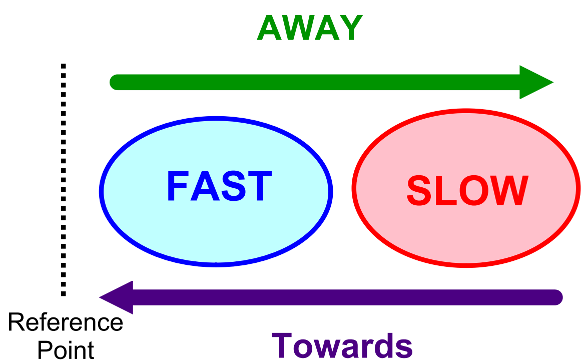


A to B	B to C
D: Towards	D: Away
V: Non-constant fast to slow	V: Non-constant slow to fast
A: to be determined by the shape on the graph	A: to be determined by the shape on the graph

Acceleration Graph Analysis
Direction / Velocity

negative acceleration

For situations when acceleration is negative (any shape in the negative domain), there will be 3 cases that apply. We know that for each case velocity will be **Non-constant**, and we must determine whether it is slow to fast or fast to slow. To determine these 3 cases we will go the cloud, the acceleration cloud. The cloud diagram for negative acceleration is shown below.

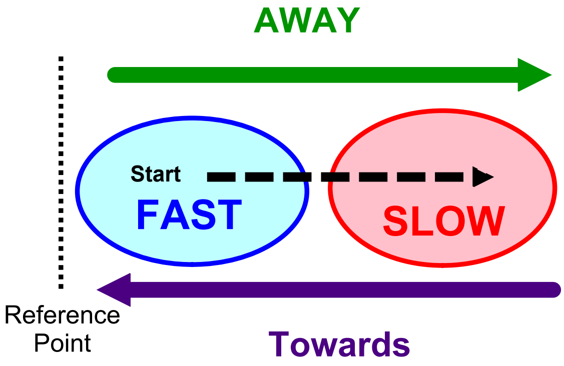


When acceleration is negative, the fast zone is near the reference point.
Note: Acceleration may be Constant ≠ 0 or Non-constant.

Acceleration Graph Analysis
Direction / Velocity

Negative Acceleration - Case 1

Choose a direction for the motion, this choice is arbitrary, but it will determine if the non-constant velocity is fast to slow or slow to fast. For case 1 the direction will be away. In order to walk away on the cloud diagram, you must start in the fast zone and move into the slow zone. Thus, the DVA description is:



D: Away

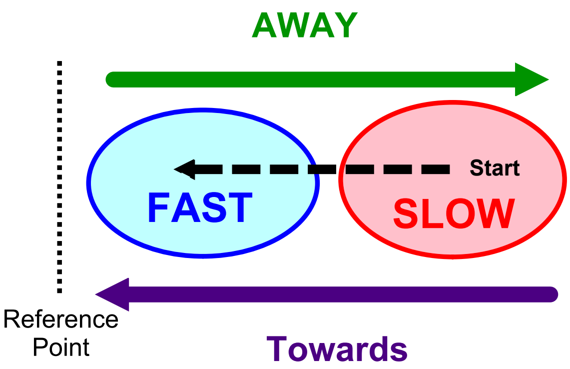
V: Non-constant
fast to slow

A: to be determined
by the shape on
the graph

Acceleration Graph Analysis
Direction / Velocity

Negative Acceleration - Case 2

Next choose the opposite direction for the motion. For case 2, the direction will be towards. In order to walk towards on the cloud diagram, you must start in the slow zone and move into the fast zone. Thus, the DVA description is:



D: Towards

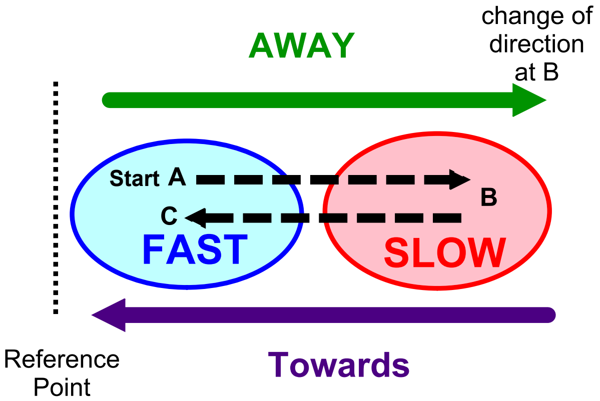
V: Non-constant
slow to fast

A: to be determined
by the shape on
the graph

Acceleration Graph Analysis
Direction / Velocity

Negative Acceleration - Case 3

No further directions for the motion can be used (Standing Still is NOT an option). For case 3, we will break the graph into an A-B-C graph, where there is a change of direction at B. We know that velocity is ZERO when direction changes, so point B must be in the slow zone. Since the slow zone is away from the reference point for negative accelerations we must start in the fast zone. Thus, A to B is walking away starting in the fast zone and moving into the slow zone. Then B to C is walking towards starting in the slow zone and moving into the fast zone. Thus, the DVA description is:



A to B	B to C
D: Away	D: Towards
V: Non-constant fast to slow	V: Non-constant slow to fast
A: to be determined by the shape on the graph	A: to be determined by the shape on the graph

Acceleration Graph Analysis
Jerk

Slope on an acceleration graph is calculated by taking the rise and dividing it by the run (rise over run). The rise on an acceleration graph is change in acceleration measured in m/s^2, the run is time measured in seconds. The slope is then:

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{change in acceleration}}{\text{time}} = \text{Jerk}$$

$$\text{units for Slope} = \frac{m/s^2}{s} = \frac{m}{s^3}$$

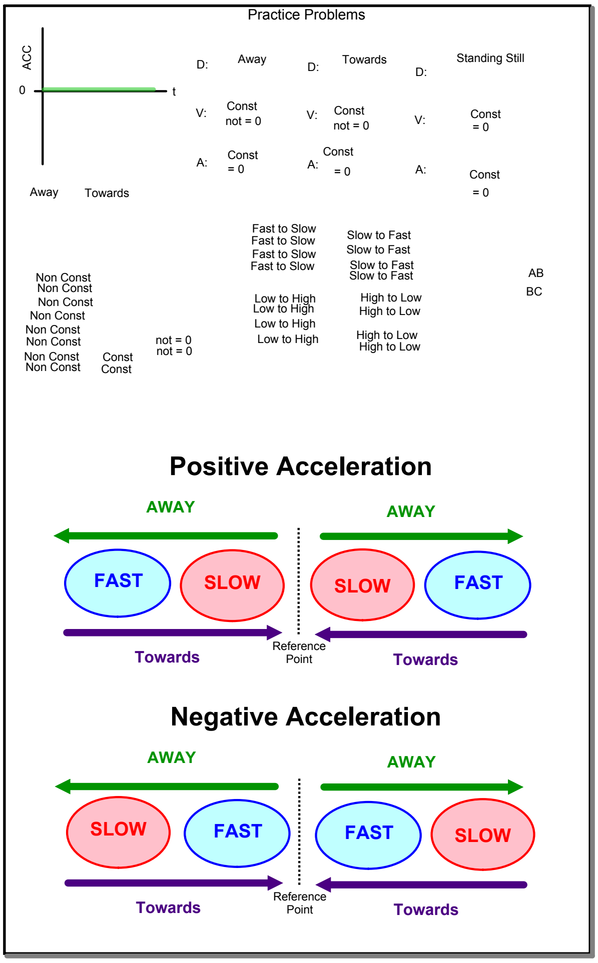
Slope on an Acceleration Graph is Jerk

Acceleration Graph Analysis
Jerk

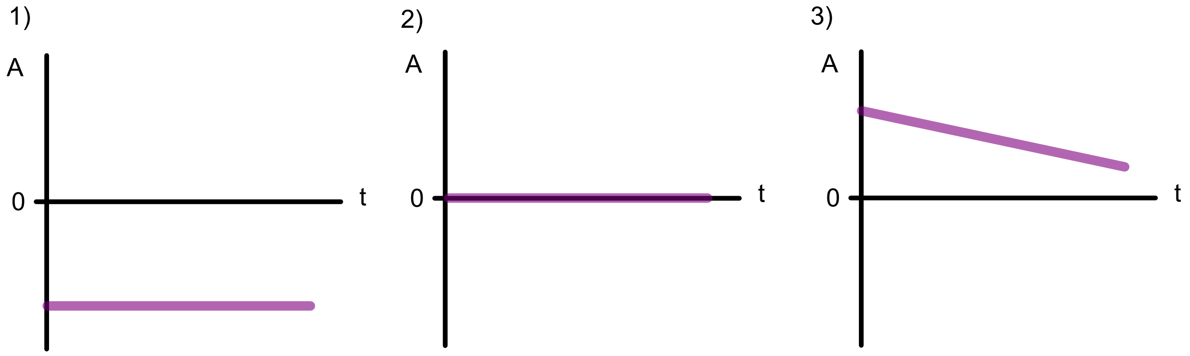
In physics, jerk, also known as jolt, surge, or lurch, is the rate of change of acceleration; that is, the derivative of acceleration with respect to time, the second derivative of velocity, or the third derivative of position.

Jerk is often used in engineering, especially when building roller coasters. Some precision or fragile objects — such as passengers, who need time to sense stress changes and adjust their muscle tension or suffer conditions such as whiplash — can be safely subjected not only to a maximum acceleration, but also to a maximum jerk. Even where occupant safety isn't an issue, excessive jerk may result in an uncomfortable ride on elevators, trams and the like, and engineers expend considerable design effort to minimize it.

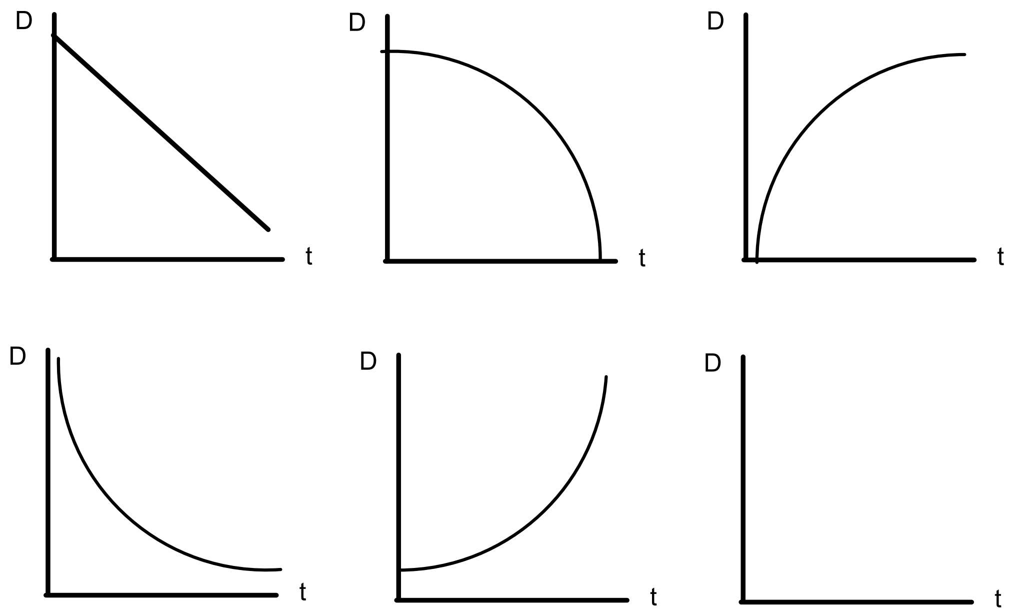
For example, two drivers, driving with an uncovered full cup of hot coffee between their legs, come to a stop sign. The experienced driver gradually applies the brakes, causing a slowly increasing acceleration (small jerk), and comes safely to a stop without spilling the coffee. An inexperienced driver (teenager) applies the brakes suddenly (slamming on the brakes because the where texting), causing a rapid increase in acceleration (large jerk). The sensation of jerk is very noticeable, causing the driver's head to jerk forward and spilling the coffee severely scalding the driver.



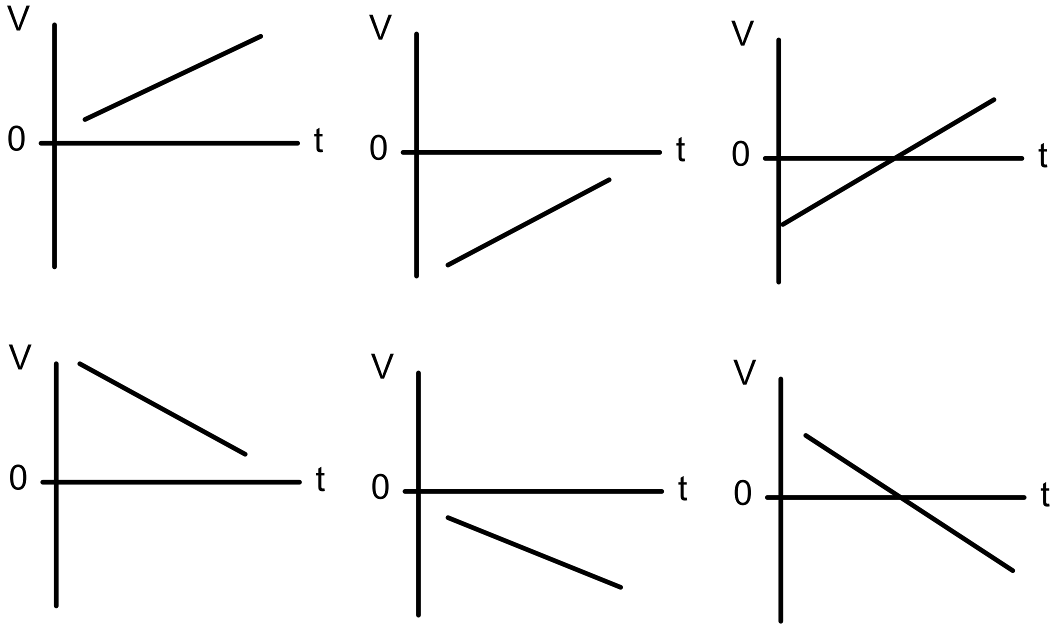
Quiz



Determine if the acceleration is positive or negative.



Determine if acceleration is positive or negative



Determine if acceleration is positive or negative