

## Distance and Displacement

Distance and displacement are two quantities which may seem to mean the same thing, yet they have distinctly different meanings and definitions.

### Distance

Distance is how far the object traveled or how far it is from its initial location.

Distance is a scalar.

Distance gives information about the actual route taken (path).

Distance gives very little information about the final position.

### Displacement

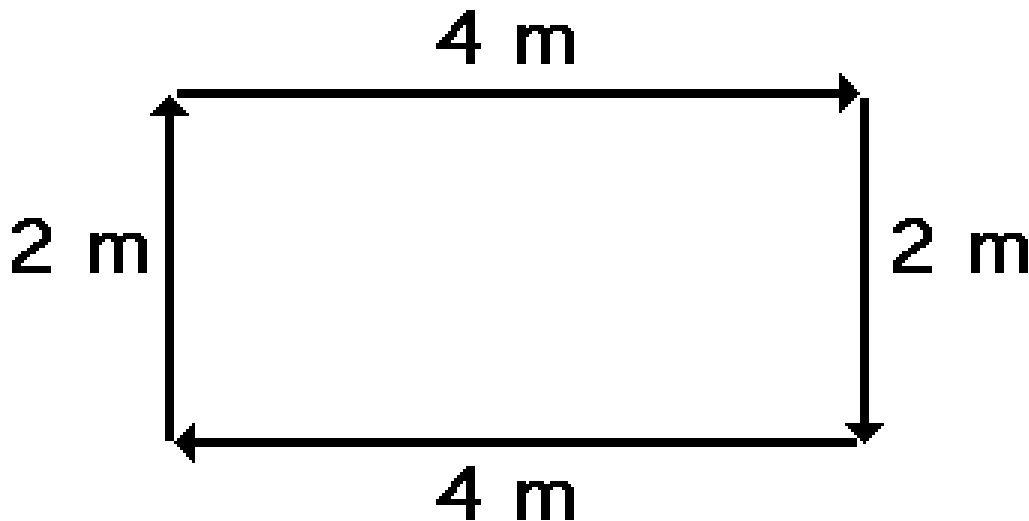
Displacement is a direction as well as a distance from the initial to the final position..

Displacement is a vector. e.g., 10 miles west.

Displacement provides the exact location of the final position of the object.

Displacement provides no information about the path the object followed.

To test ones understanding of this distinction, consider the motion depicted in the diagram below. A physics teacher walks 4 meters East, 2 meters South, 4 meters West, and finally 2 meters North.



Even though the physics teacher has walked a total distance of 12 meters, his displacement is 0 meters. During the course of his motion, he has "covered 12 meters of ground" (distance = 12 m). Yet, when he is finished walking, he is not "out of place" – i.e., there is no displacement for his motion (displacement = 0 m). Displacement, being a vector quantity, must give attention to direction. The 4 meters east is canceled by the 4 meters west; and the 2 meters south is canceled by the 2 meters north.

To understand the distinction between distance and displacement, one must know their definitions and also know that a vector quantity such as displacement is direction-aware whereas a scalar quantity such as distance is ignorant of direction. When an object changes its direction of motion, displacement takes this direction change into account; heading in the opposite direction effectively begins to cancel whatever displacement there once was.

Distance and displacement have the same size only when one considers small intervals or intervals where the direction of the motion is constant. Since the displacement is measured along the shortest path between two points, its magnitude is always less than or equal to the distance.

Last, but not least, is the subject of symbols. How can one distinguish between distance and displacement in writing? Well, some people do and some people don't and when they do, they don't all do it the same way. Although there is some degree of standardization in physics, when it comes to distance and displacement, it seems like nobody agrees.

What would be a good symbol for distance? Hmm, I don't know. How about  $d$ ? Well, that's a fine symbol for English speakers, but what about the rest of the planet? (Actually, distance in French is spelled the same as it is in English, but pronounced differently, so there may be a reason to choose  $d$  after all.) In the current era, English is the dominant language of science, which means that many of our symbols are based on the English words used to describe the associated concept. Distance does not fall into this category. However, it is acceptable to use  $d$  to represent distance.

All right then, how about  $x$ ? Distance is a simple concept and  $x$  is a simple variable. Why not pair them up? Many textbooks do this, but not all. The variable  $x$  should be reserved for one-dimensional motion along a defined  $x$ -axis (or the  $x$  component of a more complex motion). However, it is, also, acceptable to use  $x$  to represent distance.

English is currently the dominant language of science, but this has not always been the case nor is there any reason to believe that it will stay this way forever. Latin was preeminent for a very long time, but it is little used today. Still, there are thousands of technical and not so technical words of Latin origin in use in the English language. Medicine, it seems, would be without vocabulary were it not for this "dead" tongue -- cardiac, referring to the heart; podiatry, the treatment of the feet; dentistry, the treatment of the teeth; etc. Examples are less common in physics, but they are there nonetheless. (There seem to be more Greek than Latin words in physics.)

Imagine some object traveling along an arbitrary path in front of an observer. Let the observer be located at the origin. The vector from the origin to the object points away from the observer much like the spokes of a wheel point away from its center. The Latin word for spoke is radius. For this reason, we will use  $r_0$  ( $r$  nought) for the initial location,  $r$  for the location any time after that, and  $\Delta r$  ( $\Delta$   $r$ ) for the change in

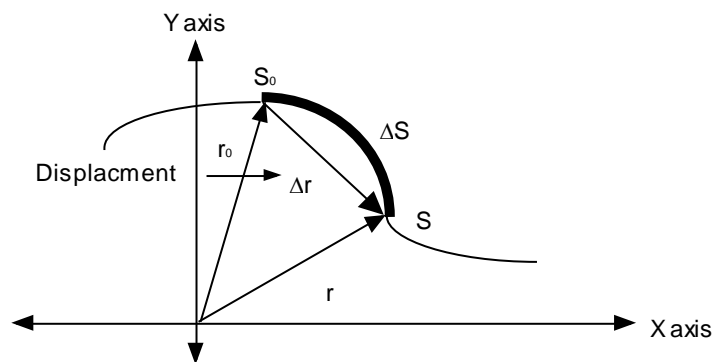
location -- the displacement. Unlike the spokes of a wheel, however, this radius is allowed to change.

Much more directly, but less poetically, the Latin word for distance is spatium. For this simple reason, we may use  $s_0$  (s nought) for the initial location on a path,  $s$  for the location on the path any time after that, and  $\Delta s$  (delta s) for the space traversed going from one location to the other -- the distance.

If Latin deserved its reputation as a "dead tongue" then no one would still use these symbols, but their use is quite common. Old habits die hard. Use of spatium goes back to the first book on kinematics as we know it -- Galileo's Discourses on Two New Sciences in 1640.

*Spatium transactum tempore longiori in eodem Motu aequabili maius esse spatio transacto tempore breuiori.*

For the same motion, with all other factors being equal, the distance traversed in a longer span of time is greater than the distance traversed in a shorter span of time.



One important thing to notice in the diagram above is that the location of the observer does not really matter. One may think that the observer must be located at the origin, but this is not the case. It is merely convenient for the sake of illustration. If the observer were not at the origin, we could always move the origin to the observer. In addition, the x-axis need not be horizontal nor must the y-axis be vertical. No matter how you twist the coordinate system, the essence of the diagram remains unchanged. Distance and displacement are said to be isotropic, that is, they remain unchanged even if the coordinate system undergoes translation or rotation. All properly formulated physical laws must be isotropic.

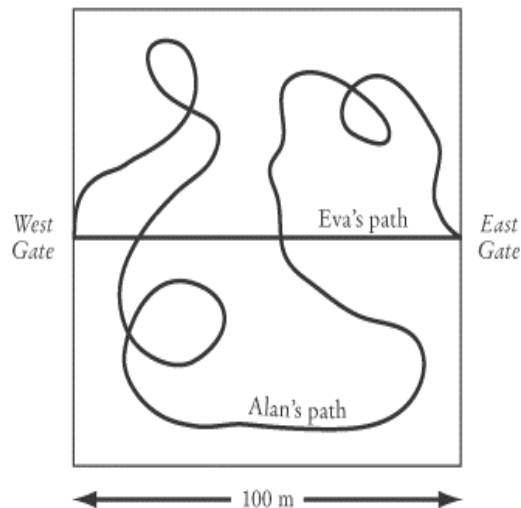
The AP convention for distance and displacement are the ones that will be the primary focus for this course. They conform to the symbols derived from Latin, sort of. They, however, change the symbols and meanings.  $r$  is used to represent radius or distance while  $s$  is used to represent displacement. For linear and non-linear motion, an  $x$ ,  $y$ , or  $z$  is used to represent displacement along that particular axis (direction). Also, they will use unit vectors  $i$ ,  $j$ , and  $k$ . ( $i$  means the  $x$  direction,  $j$  the  $y$  direction, and  $k$  the

z direction.) The following was copied directly from the SAT Physics Spark notes web page:

Displacement is a vector quantity, commonly denoted by the vector  $s$ , that reflects an object's change in spatial position. The displacement of an object that moves from point A to point B is a vector whose tail is at A and whose tip is at B. Displacement deals only with the separation between points A and B, and not with the path the object followed between points A and B. By contrast, the distance that the object travels is equal to the length of path AB.



Students often mistake displacement for distance, and SAT II Physics may well call for you to distinguish between the two. A question favored by test makers everywhere is to ask the displacement of an athlete who has run a lap on a 400-meter track. The answer, of course, is zero: after running a lap, the athlete is back where he or she started. The distance traveled by the athlete, and not the displacement, is 400 meters.



Alan and Eva are walking through a beautiful garden. Because Eva is very worried about the upcoming SAT II Physics Test, she takes no time to smell the flowers and instead walks on a straight path from the west garden gate to the east gate, a distance of 100 meters. Alan, unconcerned about the test, meanders off the straight path to smell all the flowers in sight. When Alan and

Eva meet at the east gate, who has walked a greater distance? What are their displacements?

Since Eva took the direct path between the west and east garden gates and Alan took an indirect path, Alan has traveled a much greater distance than Eva. Yet, as we have discussed, displacement is a vector quantity that measures the distance separating the starting point from the ending point: the path taken between the two points is irrelevant. So Alan and Eva both have the same displacement: 100 meters east of the west gate. Note that, because displacement is a vector quantity, it is not enough to say that the displacement is 100 meters: you must also state the direction of that displacement. The distance that Eva has traveled is exactly equal to the magnitude of her displacement: 100 meters.

Remember, these are the same people who write the AP tests, so we will use their symbols for distance and displacement.

### Units

The SI unit of distance and displacement is the meter [m]. A meter is a little bit longer than the distance between the tip of the nose to the end of the farthest finger on the outstretched hand of a typical adult. Originally defined as one ten thousandth the distance from the equator to the north pole (as measured through Paris); then the length of a precisely cut metal bar kept in a vault outside of Paris; then a certain number of wavelengths of a particular type of light -- the meter is now defined in terms of the speed of light. One meter is the distance light (or any other electromagnetic radiation of any wavelength) travels through a vacuum after  $1/299,792,458$  of a second.

Multiples (like km for road distances) and divisions (like cm for paper sizes) are also commonly used in science.

There are also several natural units that are used in astronomy and space science.

- A nautical mile is now 1852 m (6080 feet), but was originally defined as one minute of arc of a great circle, or  $1/60$  of  $1/360$  of the earth's circumference. Every sixty nautical miles is then one degree of latitude anywhere on earth or one degree of longitude on the equator. This was considered a reasonable unit for use in navigation, which is why this mile is called the nautical mile. The ordinary mile is more precisely known as the statute mile; that is, the mile as defined by statute or law. Use of the nautical mile persists today in shipping, aviation, and aerospace.
- Distances in near outer space are sometimes compared to the radius of the earth:  $6.4 \times 10^6$  m. Some examples: the planet Mars has  $\frac{1}{2}$  the radius of the earth, the size of a geosynchronous orbit is 6.5 earth radii, and the earth-moon separation is about 60 earth radii.

- The mean distance from the earth to the sun is called an astronomical unit: approximately  $1.5 \times 10^{11}$  m. The distance from the sun to Mars is 1.5 AU; from the sun to Jupiter, 5.2 AU; and from the sun to Pluto, 40 AU. The star nearest the sun, Proxima Centauri, is about 270,000 AU away.
- For really large distances, the light year is the unit of choice. A light year is the distance light would travel in a vacuum after one year. It is equal to some nine quadrillion meters (six trillion miles).

For each of the following questions, decide whether the question is asking for the distance or the magnitude of the displacement.

How far do you walk each day?

How far is it from Toronto to Mexico City by airplane?

How far is it from Toronto to Mexico City by car?

How far is the earth from the moon?

How long is a standard sheet of plywood?

How long is the coastline of Britain?

How long is the Nile?

How long is a football field?

How far did you walk to get to class?

How many frequent flyer miles do you have?

What is the radius of the earth?

How far is it to the north pole?